



**THINKING**

**STATISTICALLY**

**URI BRAM**

# Thinking Statistically

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by Uri Bram

to Elizabeth Swerdlow,  
my favourite statistician.

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# INTRODUCTION

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*It's not just you*

You're smart. Really smart. People are always complimenting your curiosity and your general knack for understanding things; maybe you even have a diploma from a fancy university, though that's irrelevant for our purposes. You're really smart, but (deep breath) you don't understand Bayes' Theorem. Or endogeneity. Or selection bias. Maybe you've never heard of them, maybe you read about them once in a textbook. You definitely don't use them in your everyday life. It's not your fault for reasons completely beyond me, rarely does anyone explain these concepts to you properly unless you happen to be studying statistical techniques. But Bayes and selection bias and endogeneity are *concepts*, not techniques; understanding them is essentially independent from knowing how to find a p-value, or the conditions for using a Student's t-distribution, or the proof of the Central Limit Theorem.

Concepts and abstractions are often useful even if you aren't exactly interested in the details underneath them. It's good to understand that if the Central Bank raises interest rates then people tend to save more, without understanding the technical details of how a Central Bank effects interest rate changes. It's good to understand that there are sets of chords that harmonise together, and that many pop-songs are based on a few simple progressions, without understanding anything deeper about music (or physics). It's good to understand the logic of evolutionary theory, even if you never plan to dust a fossil or examine mitochondrial D.N.A. Similarly, there's actually no good reason why you have to learn statistical techniques in order to understand statistical concepts. The concepts on their own can be a huge help when examining and interpreting the world around us.

Most statisticians (and econometricians, and scientists) use statistics *informally* in their everyday lives, and a surprising number assume that everyone else does too. The world is made up of information, and we make all kinds of decisions and judgments based on the information that reaches us: if that information is systematically biased or incomplete then the decisions we make will be correspondingly wrong. Formal statistics is about taking precise, detailed information and providing exactly what that information can or can't tell us; informal statistics is about taking the vague, general patterns of information that we see in everyday life and using the same basic statistical concepts to make generally better decisions and judgments. Not perfect judgments, sure, and not ones you can hang numbers on, but a better judgment is a better judgment and small improvements add up fast.

This book will teach you how to think like a statistician, without worrying about formal statistical techniques. Along the way we'll learn how selection bias can explain why your boss doesn't know how to do his job (even when everyone else does); how to use Bayes' Theorem to decide if your partner is cheating on you; and why Mark Zuckerberg should never be used as an example for anything. At no point will we do anything involving numbers. You can handle numbers just fine, if you want to, but you don't right now.

This book is for you.

*Alternative uses*

If I'm going to be honest here — and if you and I are going to do a book together then honesty

very important — I have a second hope for this book, and then a third.

~~The second hope is that the book can be useful for people taking traditional statistics courses, as a companion and refresher on the ideas behind the work they're doing. The truth is that nobody, no matter how statistically-minded they are, can consistently remember every idea they need to know and the key intuitions behind how things work. Don't ever forget it — these ideas aren't easy, and everyone has to look up the definitions sometimes. I hope that this book might provide a concise and accessible summary of some of the key concepts to people knee-deep in formal tests.~~

The third hope is that —well— Hal Varian, the chief economist of Google, has said that “the sexiest job in the next 10 years will be statisticians... And I'm not kidding.”\* I hope this book will make everyone who reads it as sexy as a statistician. Although people who choose to read this book seem to be exceptionally sexy already, so it might be hard to tell cause from effect.

# CHAPTER ONE:

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# SELECTION

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## *Buying apples*

So, it's Sunday morning (the birds in one another's arms, cats in the trees) and you're going to a market stall to buy some apples. You pick up a couple; you poke them, shake them, and check them for blemishes. If they seem pretty good then you grab another twenty and put the whole lot in your shopping bag. You have just drawn an inference (that the apples are good) about a whole population (all the apples at this market stall) based on one small sample (the first few apples).

Whether you like it or not, you have just performed something called "inferential statistics." If you were a more formal statistician you could run some fancy tests and figure out very precise things about the apple-population: for example, the probability that all the apples at the stall are good, given that a certain number of randomly-selected samples were. But for any normal purposes, the first few apples are a good-enough indicator of the quality of the rest.

They are only a reasonable indicator, though, if they are a (fairly) random selection from the population of apples as a whole. This is probably true if you chose them yourself. But what if, instead, the stall-holder saw you gazing applewards and proffered you two to poke at? Now you would have very little reason to trust that the sample-apples were representative of the general population: if the stall-holder is wily, and if she doesn't expect to see you again, then it's plenty likely that she uses her best apples as samples and so hides the fact that the rest are pretty bad. In statistics, this issue has the snazzy name of Selection Bias: basically, that your inferences will be biased if you use a non-random sample and pretend that it's random.

## *Self-biasing data*

In the apple example, the source of bias is obvious — the stall-holder's nefariousness. If her trick succeeds it's because we aren't paying attention; with a moment's pause, nobody would believe that the stall-holder's choice is even vaguely random. But often it's much harder to tell a good (informative) sample from a bad one, partly because data samples can inadvertently bias themselves. There are lots of valid things we can say about a population from a smaller sample of it, *if* that sample is truly random. There are even lots of valid things we can say about a population if our sample is biased in some systematic way that we understand and correct for. The real problems occur when our sample is biased and we fail to account for that.

Imagine, for example, that you go to a daycare centre to take a survey of the children's ages. Your sample is the children who showed up that morning, and your population is all the children who ever attend that daycare. So far so good: some children will be missing today, but assuming it's a normal morning there's no reason to think that the missing-ness will relate to age. Then you shout for all the one-year-olds to please stand up and be counted, then the same for all the two-year-olds, and finally the three-year-olds. Any researcher would notice here that not all the children had stood. But a biased researcher would assume that the missing datapoints were missing at random, and could safely be ignored.

Now, any survey which can't absolutely coerce answers from every desired respondent will inevitably face some degree of non-responsiveness, and so long as the missing datapoints are genuinely missing-at-random it really is safe to ignore them. However, in our daycare example there's a clear possibility that the data is missing dependent on the value it would have taken. Younger children are quite liable to be under-represented in our sample, since one-year-olds are less likely to understand our instructions, less likely to co-operate even when they do understand, and more likely to be distracted by a particularly interesting bug. Our sample is biased towards kids who've got their heads together.

#### *President Truman and the census*

You might think that specific example is fairly silly, which admittedly it is. But it represents a genuine problem with sample selection that plagues even professional statisticians. Perhaps the most notorious example of selection bias in public surveys is Democrat Harry Truman's "upset" electoral victory in the 1948 U.S. Presidential election, which every pollster had predicted he was going to lose by a landslide. Where did the pollsters go wrong? They had based their numbers on a series of telephone surveys — probably the most extensive, expensive and scientific telephone survey that had ever been done — and which confidently predicted a Republican victory. But who owned a telephone in 1948? Well, generally wealthier people. Who happened to lean Republican.\*

As another example, the U.S. Census Bureau has perpetual problems calculating the nation's population, despite a tremendous effort and the power of government behind them. And we're not talking a couple of missing people: after the 2000 Census, the Bureau estimated that 4.5 million people were missing from its count, largely low-income Blacks and Hispanics.\* For all kinds of reasons, the probability that a person will fail to return her census forms is related to her other attributes: her socio-economic status, her family situation, her location, her education levels, perhaps her ethnicity. The people at the Census Bureau are fully aware of these issues, and make heroic efforts both to chase up non-responders and to publicise the shortcomings of their final counts. But the non-responses will inevitably occur, and occur in a strongly non-random way. This is not just an academic problem: the census numbers are supposed to determine the funds that local governments receive from the federal government for social services. If residents of low-income urban neighbourhoods are unusually likely to go missing from the census, low-income urban neighbourhoods will receive less funds than they truly deserve.

#### *The feedback effect*

It's safe to assume that you aren't planning to run a major presidential poll or national census in the next few weeks, but similar effects haunt our everyday lives. For example, "feedback systems" in schools, companies, charities, and governments often suffer because the probability of a subordinate's opinion actually reaching the authorities is strongly related to whether the person's opinion was positive or negative. Bosses, donors, and academic deans often have impressions about the organisations that seem utterly wacky to their less-pleased subordinates. But when the boss's knowledge is based on the feedback that has been volunteered to them, and if they fail to correct for the bias in this sample, it's little wonder that they reach incorrect inferences. In many work environments, where there are real or perceived dangers to expressing a negative opinion about the boss' previous decisions, the kind of person who is most likely to volunteer her opinion is someone



who feels positively about the issue under discussion. Many bosses are aware of the issue in theory but it is surprisingly rare to find one who accounts for the effects of selection bias on the feedback she herself receive—and even rarer to find one who corrects for it. In some ways, this explanation of why superiors can be so delusional is rather comforting. It posits that many instances of supervisor madness are not the result of malice or idiocy, but rather genuine unawareness due to impeded data flows (and resultant sample selection bias).

If you find yourself in a supervisory position, you can outsmart and outperform your rivals by putting in place systems that *genuinely* ensure that the probability of feedback reaching you is not dependent on the values that feedback takes. There are many possible systems that could work. Anonymous comment-boxes are the right kind of idea: by disconnecting ideas from names, they stop subordinates from expecting reward or punishment depending on the value their feedback took. However, there are often doubts about quite how anonymous an anonymous system is, and it is difficult as a supervisor to weigh the negative views on slips of paper as strongly as the positive views you hear in person. As a result, the best system is probably one that involves encouraging negative feedback in person: finding subordinates you trust and asking them what *other* people complain about most; encouraging a “Devil’s Advocate” mentality during projects and meetings; or asking subordinates what the least-best thing about a particular project is (“yes I know you think this project is wonderful. But if you *were* to think it wasn’t wonderful, what would you think wasn’t wonderful about it?”) Ultimately, your biggest strengths will be your simple awareness that negative feedback must exist in your organisation, and your determination to keep looking for it despite the psychological temptation for positives to be reinforced.

When you’re in the position of the supervised, rather than the supervisor, you can remind yourself of the importance of raising relevant issues with supervisors even when the issue seems obvious, when you’d previously assumed that if nothing was being done it was because the higher-ups didn’t care. In the best case they may be willing to re-examine the issue once made aware of the rest of the evidence, but at the very least you can have the comfort of realising that you’re not being treated with purposeful malice. Understanding statistics can be an unexpected source of inner peace.

### *Forgetting friends*

By contrast, the following advice may have a negative effect on your self esteem— sometimes noticing sample selection bias can knock your sense of self importance. Just don’t say I didn’t warn you.

When I first got to America I noticed that a lot of people seemed to remember me long after I’d forgotten them. This led to all kinds of awkwardness, since they knew my name and where we’d met while I had no idea who they were. But it wasn’t fair, I thought: of course they remembered me— I was the guy with the curious accent, and they were just one of the hundred identical Americans (sorry, Americans) that I’d met that month.

I mentioned this odd phenomenon to a religious Jewish friend. “I have the same thing!,” he said. “everyone remembers the guy with the Jewish head-covering, but how am I meant to distinguish among these identically-dressed hipster girls?” A short-haired female friend had a similar response: “I totally get you,” she answered, “everyone remembers the short-haired girl, we’re pretty rare here, but there’s no way I’m going to remember them as well. It’s not fair, it’s just not fair.”

By this point I was hugely suspicious that, in fact, all three of us had in fact over-estimated how memorable we were relative to other people: on reflection, all three of us had fallen victim to sample selection bias. The problem was two different kinds of missing data.

First, in my own interactions, I lack a lot of data about the times when other people forget me. ~~Everyone has (presumably) had moments where they couldn't remember someone's name from Adam, but managed to fake through a short conversation by just saying "you" or "buddy" or "heyyy".~~ When that data has the value "I forgot your name, you remembered mine" then the data always reaches me, in the sense that I'll always know whether I remembered your name or not. But there are lots of instances when someone else forgets my name completely, spends a five-minute conversation calling me "buddy," and I just didn't notice anything. Round one to sample selection bias.

The second class of missing data involves the endless interactions that don't include me at all, in which one party forgets the other party's name: where my religious Jewish friend, or my short-haired female friend, forgets the name of someone who remembers them. Out of the total population of forgotten-name incidents (the ones where I forget someone's name; the ones where someone else forgets mine; the ones where Person A forgets Person B's name; the ones where Person B forgets Person A's), the last group is almost completely invisible to me. Except for in very exceptional circumstances, I am never going to be aware of situations in which someone else forgets another someone else's name—even though, intellectually, it's clear that this must (often) happen. Round two to sample selection bias.

The data that reaches me, then, is overwhelmingly skewed towards the instances where someone remembers me and I don't remember them. Due to my biased sample, I wrongfully draw the inference that I'm unusually memorable (yes, yes, it's hard being a celebrity). Now, presumably some people forget names more and some people are forgotten more; it's possible that you, personally, really are so unique that everyone remembers you while you can't remember them. However, it's important to realise that sample selection bias could trick you into feeling this way even when it isn't actually true.

### *Congratulations*

There are actually many areas of personal life where selection bias occurs because we only experience "first person" the things that happen to us, and don't always notice that we don't receive the same stream of data from other people's first-person experiences.

When I was younger I learnt (very basic) guitar. I played mainly in my room, in the class "awkward teenager croons melancholy love-songs" mold. Sometimes, feeling plucky, I even did Open Mic Nights. And invariably, after the show, people would come up to congratulate me and say how much they enjoyed my performance—I hate to brag but some even said I was the best in the whole show. The first kind of selection bias here is obvious: if someone thinks you played badly, they're not too likely to tell you to your face. Almost-nobody comes up to a performer at a friendly open mic and says "Hey!, I saw your performance and I just wanted to say that you're a mediocre singer and barely-competent guitar-player," despite the fact that many open-mic guitarists are indeed mediocre and barely competent. Out of the entire population of opinions about my playing (positive opinions and negative opinions), the sample that reaches me is strongly biased towards the positive ones.

The second kind of selection bias is a little more subtle: immediately after a performance, you tend to drift away from everyone else, and don't get to count how many times each of the other performers gets told how wonderful they were. Intellectually you know this is happening, but pragmatically you forget about it. So out of the entire population of opinions about open-mic participants (the opinion that you were good; the opinion that you were bad; the opinion that someone else was good; the opinion that someone else was bad) you'll hear a lot about how good you were, almost-nothing about how bad you were, a decent amount about how bad other performers ("some of those guys shouldn't be on a stage at all, they're so barely competent") and maybe a little about how

good some other performers were (“that girl just before you was amazing! I mean, I liked yours more obviously. But, y’know, she was ok.”) The sample of opinions that reaches you is massively biased and so you will greatly over-estimate how much other people liked your music and under-estimate how much they liked other performers’. Of course the other performers’ biggest fans aren’t talking to you right after the show - they’ve gone to talk to their own favourite musician. As a consequence, you over-estimate how good you were compared to other performers.

Constantly being aware of the extent that selection bias affects our self-perception would be disastrously depressing (I did warn about that earlier, right?) Often it’s probably better for us to ignore the sample selection bias, enjoy strumming on the big stage, and gracefully accept the compliments afterwards. But if you get to the point where it feels like *everyone’s* telling you you’re an amazing musician and you could *definitely* make it if you went professional, it’s probably good to stop and think about the selection bias in your feedback channels. *Before* you quit your day-job.

# CHAPTER TWO:

# ENDOGENEITY

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*Things cause other things*

“Endogeneity” is one of those words so thoroughly ignored that it doesn’t even make it into most normal spellcheckers. This is a minor outrage, because endogeneity is one of the most useful concepts in all of statistics. It is also impossible to describe succinctly. The simple (and not necessarily helpful) definition is that something is endogenous if it is determined *within* the system of interest. By contrast, something is “exogenous” if it is determined by factors *outside* that system.

In biology, something endogenous originates within the organism, tissue or cell you’re looking at; something exogenous originates outside it. The same idea applies in social science (and in everyday life), except there is no perfect equivalent to cells and tissues and organisms. But social scientific arguments are composed of models, and real life is composed of informal, implicit models: when we think about whether one thing does or doesn’t cause another, we are creating an implicit mental equation with “the things doing the causing” on the left-hand side and “the things that are caused” on the right. These models are, in fact, the relevant analogues for cells or organisms in biology, and if we construct the models poorly then we run into all kinds of problems. For example, perhaps our model says that X causes Y when in fact Y causes X, or perhaps it claims that A causes B when in fact C causes both of them. Both of these are forms of endogeneity problem, for a precise reason that will be discussed later. For now, we can think of it like this: if our model claims that X causes Y, when in fact Y can cause X, then “the fact of Y causing X” is an important phenomenon that is occurring *outside* the logic of our model (this one is a little complicated); if our model claims that A causes B when in fact C causes both of them then C is an important phenomenon from outside our model is influencing the outcome (this one is much simpler).

If these ideas sound familiar, perhaps that’s because you’ve heard the famous maxim: “correlation does not imply causation.” Well spotted, cunning reader: correlations masquerading as causations are indeed one type of endogeneity problem, and we’ll discuss those later in the chapter. To understand endogeneity, though, we must first understand those informal, implicit mental models I mentioned. So here we go.

In real life, when we see an output of interest, we build a kind of model in our heads to explain how it occurred. The questions that we need to answer are basically, “Which inputs went in to cause this output?,” and “How much influence did each of those inputs have?” The problem could be something like, “Why doesn’t Little Mikey have any friends?” The output of interest is “Number of friends that Mikey has,” which in this case is zero, and the inputs could be things like “Little Mikey keeps punching other children in the face,” which has a very strong effect on the output, and “Little Mikey has this really annoying way of sneezing,” which does have some impact but not nearly as much. The fact that Input A has a large negative effect on the Output (number of friends) implies that if we could reduce Input A, we would increase the Output accordingly; if we reduced Input B, which has a smaller negative effect on the Output, then the Output would increase but not by as much.

As will be clear from the above paragraph, an implicit model can be written out in long-form, but it’ll quickly get lost in the details and it’s hard to convey the model with precision. Even in our simple model of Little Mikey’s popularity—two inputs and one output—was hard to keep track of who

written in prose. There is a solution, but it's often less popular than Little Mikey: equations. When Stephen Hawking was writing *A Brief History of Time* (a life-changing read, by the way), he was so afraid that every equation he put in it would cost him half his readership. He limited himself to one and the book sold 10 million copies, so if the warning was correct then that was a pretty expensive equation. Unfortunately there's no way I can write the next section without using equations, and lots of them. They'll be nice ones, I promise; when it comes to equations I'm as bad as anyone, and when I see them in a paper I tend to skip over as fast as my eyes will carry me. But since we all do use them (implicitly) in our everyday lives, and since they never seem to bother us there, they can't possibly be as scary as they look.

#### *A simple implicit equation*

Suppose you're standing in the supermarket and trying to decide which line to join. You see a couple of families with trolleys full of food, and a couple of lonely singles with a small basket each. You make a quick mental calculation: each of the families will probably take 10 minutes to clear the line, each of the singles will probably take 5. Congratulations! You've just created an implicit mental equation. It looks like this:

$$10f + 5s = t$$

*f* = # of families in line in front of you

*s* = # of singles in line in front of you

*t* = time you'll have to wait to reach the front of the line

That's not so bad, right? On the left of the equals sign we have some inputs, and on the right of the equals sign we have an output, and we can predict the output using some function of the inputs. We call the output the "dependent variable" because it can't vary freely within our model: its value is *dependent* on the values taken by the inputs. Which makes sense, because the whole point of our model was to predict the value of the output (time spent in line) based on the values of the inputs (number of families or singles in front of you). Similarly, we call the inputs "independent" because their variation cannot be determined by any of the other variables in the model. Like teenagers, independent variables won't let anyone tell them what to do or what to be.

Now, obviously there's something missing here: those "10 minutes" and "5 minutes" figures were only guesses, they weren't some kind of (impossibly precise) measure of exactly how long each person will take. We need to add something called an *error term*. The error term sweeps up all the random variation and represents it as a single letter. We can now write the proper implicit equation – again, you're doing something like this in your head every time you see a shopping line, whether you realise it or not.

$$10f + 5s = t + e$$

*f* = # of families

*s* = # of singles

*t* = time you'll have to wait

*e* = the error term

This equation tells us that if there's 2 families and 3 singles in line in front of you, you can expect to wait  $(10 \times 2) + (5 \times 3) = 35$  minutes, *plus-or-minus* the random error term. Few things in this world can be predicted with absolute certainty, and we know that some of the families will only take 10 minutes while others will take 12; some of the singles will only take 4 minutes and others 6. Often those random errors will cancel out—if someone takes a minute more than expected, and someone else takes a minute less, then you're back on track overall—but if you're unlucky you'll get stuck in line where *everyone* is slower than expected. However, you know that the average total wait time will be 35 minutes (assuming, of course, that your original equation was well-designed), and that any variation around that will be random. While there is some probability that the wait will be longer than 35 minutes, there is exactly the same probability that the wait will be shorter, and small differences from 35 minutes are much more probable than large ones. Now we have a proper estimate of the time we'll wait in line: 35 minutes, give or take something, but probably not much. Right?

*Beware the error term, my child*

At this point you may be sitting back smugly, wondering what all this error-related fuss is about. That was easy! But error terms are sneaky little monkeys, they start making problems when you least expect it. In the shopping-line equation we designed, the error term was *uncorrelated* (as it should be) with the independent variables — the random factors swept together in the error term were completely unrelated to the independent variables (the number of families, the number of singles) that we were using to make predictions. This was why we could use the formula to predict how long we would have to wait in line—we knew that, while there would be some variation from the expected time, it would be random variation and there would be no way to predict it from the information available to us.

We assume, by definition, that the error term is random variation — that errors are clustered around zero, that under-estimates are as probable as over-estimates, and that small errors are more probable than large ones. A model with random error in it is still a useful model — not quite as good as a perfect one, sure, but still worth a lot. Knowing that the shopping line will take somewhere near 35 minutes is a very good thing, even if you also know that you could easily wait 5 minutes more or less. But what happens if the error term secretly includes factors that are not, in fact, random? If there exists any variable that is correlated with the error term then the error term is non-random — this is necessarily true, basically because of what it means to be truly random. This means that the differences between our predicted outcome and the true outcome will be systematic, not random: if you had some data about the variable that was secretly correlated with the error term, you would then be able to predict in different situations whether the model would overestimate or underestimate the output.

But if our model is wrong systematically, not randomly, it is not very useful for making predictions. Think again about our model of the shopping line; suppose we see a single line with only singles in it. Our model claims that each single will take 5 minutes to clear the line, so in 20 minutes of time we'll get to the front—give or take some random error. However, in real life, this is not quite right—shoppers don't distribute themselves randomly into lines. Why are there 4 singles, one after another, in a single line? Perhaps we are looking at the special "10 items or fewer" line, which is why no families joined it. In this case, these specific singles will probably all take less than the 5 minute average, and our model will *systematically* overestimate how long we'll wait if we join this line.

As a rigorous definition, a variable is endogenous within a given system if it is correlated with the error term in the equation. Why? Because our equation is supposed to give us two things: an output and an error term. Our output should be predicted by our inputs, and the error term should sweep up

all the sources of random variation that can make individual observations vary from that predicted value. The error term, then, should be determined by random factors completely outside the system—luck, the weather, rolls of the dice. If, instead, the error term is being (partly) determined by a variable inside the equation, our model's predictions will be systematically off. The variable in question is called “endogenous”—from the Greek word *endon*, meaning ‘within,’ and possibly the Greek root *genesis* meaning birth—because it is (illegally, improperly) helping to determine the error term from inside our system.

#### *Omitted variable bias*

One way to destroy a model through endogeneity is to leave out an important variable that is needed to explain the outcome in question. Any part of your outcome that would have been explained by this missing variable will have to be swept into the error term, which will now be non-random; it will contain the entirety of the omitted variable. For example, the formula for the area of a square (of course):

$$\text{height} \times \text{width} = \text{Area}$$

where height = width. But imagine that your model of a square was simply:

$$\text{height} = \text{area} + \text{error}$$

This is a monumentally stupid formula for the area of a square, but let's run with it for now. If you believed in it you would measure a bunch of squares and notice that your formula did have some kind of truth. You'd see a strong correlation between height and area, and that the higher a square was the bigger its area tended to be. You'd predict that a square of height one should have area = 1, which is correct; that a square of height two should have area = 2, when in fact it has area = 4; and that a square of height three should have area = 3, when in fact it has area = 9. The error in your predictions is completely non-random: the error gets greater as the height gets greater, for a start, because the error “contains” the influence of the omitted variable “width.” Furthermore, for any square with area greater than one, your prediction is always an under-estimate. If you gather a large set of squares then your error terms will not average out; your formula will systematically under-estimate the areas of squares.

#### *Why G.P.A. is stupid*

A very important example of a model whose stated purpose is destroyed by omitted variable bias is college G.P.A. In the U.S., grades tend to be calculated on a 0-to-4 scale where an A-grade is worth 4.0, an A- is worth 3.7, a B+ is worth 3.3, and so forth. A student's overall G.P.A. is calculated as the average of her scores in all the courses she took during her degree. The reasons that G.P.A. is stupid, rigorously, are the exact same reasons that G.P.A. seems stupid intuitively: college students get to choose their own courses, and the difficulty of getting an A relates to the difficulty of the courses you're in, so by choosing easier courses you can get better grades.

As a freshman and sophomore I didn't worry too much about G.P.A. because I assumed no-one (not employers, surely not grad schools) would be stupid enough to assign any meaning to a metric

which so blatantly failed to capture what it claimed to. But hey, apparently nobody is immune from Old Man Endogeneity's wily tricks.

When people treat G.P.A. as if it actually captures something meaningful — for example, when they create a G.P.A. cutoff for a job posting or a grad school application — they are implicitly assuming something like this:

$$X(f) + Y(b) = \text{G.P.A.} + \text{error}$$

$f = \text{effort}$

$b = \text{ability}$

$X$  &  $Y$  are functions

The equation claims that G.P.A. is largely determined by effort and ability, plus or minus some random variation. For an individual student, the error term here would sweep up such factors as whether she got woken up by a parrot at 4 a.m. the night before an exam, or whether her laptop died in the middle of writing her term paper. These causes of variation are not correlated with a student's ability or effort. If this model were correct then college G.P.A. would still not be a perfectly 'fair' measure, on an individual level, since some students would just be unlucky and get lower scores than their effort and ability merited. For all the students in the system put together, however, it would be true that a high-G.P.A. student would with high probability possess better ability and effort than a low-G.P.A. student.

Once students are allowed to pick their own courses, however, G.P.A. becomes an implausible measure of effort and ability. While it may remain true that, for any given course you take, your grades will correlate fairly well with the effort you put in and your ability at that subject, the fact that you can choose your own courses introduces an opposing force: if you choose easier courses you can exert less effort and still attain better grades. As such, G.P.A. could correlate *inversely* with effort.

This, then, is the root of our endogeneity problem. Our error term (the difference between the G.P.A. values predicted by our first model and the actual G.P.A. outcomes) is not at all random. In fact, it is highly correlated with an omitted variable — difficulty of courses chosen. For any given level of effort and ability, taking easier courses will result in a higher G.P.A. Note that even if there are brilliant, hard-working students who get 3.8s despite taking difficult courses, they would've (generally) done even better if they had taken easier courses.

Taking into account course selection, we can see that a more accurate equation for G.P.A. would look something like this:

$$X(f) + Y(b) + Z(c) = \text{G.P.A.} + \text{error}$$

$f = \text{effort}$

$b = \text{ability}$

$c = \text{easiness of courses selected}$

$X, Y$  &  $Z$  are functions

This is still an over-simplification: there are a million other factors that determine G.P.A., from the all-important skill called "knowing-the-system" to the undoubtedly-influential "whether or not a student already has a job-offer by senior year." But our improved, three-input equation at least shows that a high G.P.A. can be caused not only by high ability and effort, which are admirable, but also by purposefully taking easy courses, which is the-opposite-of-admirable. Someone who understands this



would be reticent to draw any kind of inference from a student's G.P.A. alone; she'd see that the signal given by G.P.A. is worse than useless.

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### *Causality loops*

A second kind of endogeneity problem occurs when cause and effect are connected by a kind of "causality loop." Endogeneity will be a problem whenever the output that you're trying to explain might also be the cause, rather than the effect, of the input that you use to explain it. When your mother tells you she shouts at you because you never come to visit her, you might retort that you don't like to visit because she's always shouting at you. "Mother," you tell her, "my visits are endogenous to how much you shout at me." Her implicit equation is:

$$A(1/v) = s + \text{error}$$

$v$  = number of times you visit,  
 $s$  = number of times she shouts at you,  
 $A$  = some function of  $1/v$ .

In her model, the number of times she shouts at you is inversely related to the number of times you visit: the more you visit, the less she shouts. But you point out that all her shouting causes you not to visit:

$$X(1/s) = v + \text{error}$$

In your model, the number of times you visit is inversely related to the number of times she shouts at you. So the amount you get shouted at is endogenous to how much you visit, and the amount you visit is endogenous to how much you get shouted at. The upshot is that we can't draw any reliable inferences about how much you will visit (or be shouted at) from the models written above; we'd need to create a better model that could handle the endogeneity problem properly.

### *People who switched*

A similar problem plagues insurance advertising. Insurance firms regularly boast that "people who switched their insurance to us saved an average of \$102!" This tidbit tells us exactly nothing about the probability that we would save money by switching to their insurance. The sample of people who switched insurance is massively endogenous; people who discover that they'd save a lot might switch providers, but people who discover that the new company would be more expensive for them are probably going to stay put. In a world where two insurance companies offered exactly the same average prices, with a random assortment of people finding that they could save money by switching from A to B but the same number able to save just as much by switching from B to A, *both* companies could honestly advertise that "people who switched their insurance to us saved an average of \$X!" In terms of endogeneity, the outcome to be explained (saving money) is actually a cause of the insurance firms' explanation of that outcome (switching insurance providers), so the inference they unsubtly imply (that you, or any other random person, will probably save money by switching to their insurance) is completely false.

In some ways we should be more impressed by a company that advertised that “people who switched their insurance to us paid an average of \$40 MORE.” This implies that people found the product so excellent that they were willing to pay more to get it, which at least makes it worth spending some time to find out more about it.

*Bill and Mark are endogenous*

Endogeneity is also the reason that you should never trust anyone who tells you something like “Bill Gates and Mark Zuckerberg both dropped out of college and did very well for themselves; therefore, we need to encourage our young-folks to be more adventurous, risk-taking, and entrepreneurial.” Bill and Mark were not just two randomly-selected Harvard undergrads — first, the fact that they dropped out of college was endogenously determined by their personalities, which also affected their probability of success; second, the fact that they are known about by the person giving the example is itself endogenous to their later successes.

For Bill and Mark, part of the endogeneity was from *personality*. What kind of person drops out of Harvard aged 20? Well, suffice to say that it’s not an ordinary Harvard undergrad, given the graduation rate. In a world where dropping out of college is a big taboo, and where going to Harvard is a kind-of-a-big-deal, the sort of person who drops out of Harvard is not an ordinary sort of person at all (or an ordinary sort of Harvard undergrad, which is far from the same thing). Bill and Mark probably both had some extraordinary personality traits — boldness, courage, hacking ability, self-belief — which helped them greatly in their business careers, *and* which made them willing to drop out of Harvard.

In Mark’s case there was an additional source of drop-out-decision endogeneity: he already had a pretty successful company, with Silicon Valley offices and Peter Thiel’s investment, before he officially dropped out of school. The omitted variable — whether or not your company is already mind-blowingly successful — is surely correlated to the probability that you’ll decide to drop out of college, and your decision to drop out of college will then seem to have more of a positive influence on your successes than it really should. People who just note that Mark was a risk-taking college dropout, but fail to note that he was already massively successful before he dropped out, will overestimate the benefit of dropping out for those *without* an already-successful company.\*

The first type of endogeneity problem, discussed above, occurs at the level of deciding to drop out. But there is a second endogeneity problem that occurs at the level of people-discussing-college dropouts. Most college dropouts are not well-known enough that strangers would discuss them when talking about college dropouts. Imagine if your Cousin Frank abandoned Harvard and went on to live in his girlfriend’s garage for the rest of his life, running a failed internet startup. *You* would know about Cousin Frank’s misadventures, and the rest of Frank’s family would, and some of his high school friends. Let’s say, generously, that 1,000 people would be aware of Frank’s dropout status and his unsuccessful existence (maybe through reading his Facebook updates). By contrast, millions and millions of people know about Mark Zuckerberg’s story, how he dropped out of Harvard and became a billionaire: 20 million officially saw the Zuckerberg biopic *The Social Network*, and 500 million have accounts with his website. Taking the low-ball 20 million figure, for every one Mark Zuckerberg there could be 20,000 Cousin Franks, and (if none of the Cousin Franks had any mutual friends, which is a stupid assumption but let it slide for now) then the 20 million people who knew of one Mark Zuckerberg and one Cousin Frank might assume that college dropouts had a 50/50 chance of becoming billionaires. Not bad odds, I’d say. But of course this calculation is junk: the ‘actual’ odds (in our artificial example) would be 1 / 20,001. Not so hot.

If endogeneity can fool ordinary people, it can also apparently fool huge multinational companies. A leading American management-consulting firm has a prominent graph on its website showing how their clients' share-price gains since 1980 have outperformed a stock-market index by a factor of four. This looks pretty impressive, unless you think about it. There's two endogeneity effects here. First, the kind of client who tries to hire a consultant is not representative of companies as a whole — perhaps the companies who hire consultants have more dynamic leadership to begin with; perhaps the future is looking bright for them, and they feel that they have cash to spare; or perhaps they were doing unusually poorly, needed restructuring, but were about to bottom-out anyway. Second, the clients that a consulting firm accepts are not a random sample either: a consulting firm may only take on clients that it thinks it can help, or perhaps even that it thinks will have share-price gains in future years (opposed to one that will still do badly, but perhaps less-badly, with the consultants' help). If the consulting firm had randomised which clients it accepted and which it didn't, the difference in share-price gains between the accepted and rejected firms would provide a meaningful gauge of the consultants' contribution. The statistics presented on the website, however, tells us more about who hires consultants than about how much consultants help companies.

*Social science and natural science*

One of the great problems of social science is that nothing ever sits still. Modern statistics got its big break with 1920s agricultural experiments: you take two nearby rows of corn and treat them with two different fertilizers, and at the end of the season you see which field grew better. The early years in the development of statistics were largely about figuring out the quality of inferences you can make from different kinds of data: how do you tell if the different yields were caused by a better fertiliser or just a fluke? How many ears of corn do you need to check before you can confidently declare the characteristics of the entire field? These are difficult questions, and it took some obscenely smart people to provide rigorous answers. But one comfort we had was that the direction of causality was completely clear. The independent variable (which fertiliser you used) would affect the dependent variable (how much corn grows), but there was absolutely no way that the dependent variable (corn growth) could influence the independent variable (which of the fertilisers you had already used on it).

With humans involved, nothing is ever so simple. Suppose you're interested in primary schooling in developing countries, and want to see the effects that donating textbooks has on the students' educational outcomes. You find a village with two identical schools in it and give textbooks only to the students in School A. So far, so good: you have an independent variable (whether a student got a textbook) and a dependent variable (the students' educational outcomes at the end of the study). But unlike corn-stalks, children can *move*. If they (or their parents) start to realise that one school is getting better resources than the other, some of the more ambitious students might try to move across. The end of the study arrives and the students in School A achieved better academic outcomes. But did the textbooks cause the better outcomes, or did the more ambitious students cause themselves to get given textbooks? How much of the gains are due to each cause? It's hard to tell. The supposedly dependent variable has started influencing the supposedly-independent ones. Cause and effect have become difficult to disentangle.

In fact, endogeneity plagues all kinds of social science research, and many of the social sciences

questions that make it into the news. If you see a piece of social science reported in the media, one of your first questions should probably be “is there an endogeneity problem here?” This is not necessarily the first question you should ask about social science research *in general*, but there seems to be a kind of endogeneity problem with the kind of social science that gets reported in the media, namely that the kind of papers that make it into the news are unusually likely to suffer from endogeneity problems. To a stats nerd, this passes as pretty funny.

*Not all that correlates is caused*

If there’s one statistical maxim which is widely known among non stats-nerds it’s “correlation does not imply causation.” *Correlation* means, roughly, that two variables are inter-dependent: if one goes up, the other goes up with it. (If two variables are *inversely correlated* then as one goes up, the other goes down). *Causation* means, well, that one *caused* the other to happen. But two variables can be correlated without any causal relationship between them: it could just be coincidence, or it could be that they are both caused by an invisible third factor. Just because two things vary together does not mean one caused the other.

A classic example of “correlation does not imply causation” is the famous story that ice-cream sales over the course of a year tend to correlate with the number of drownings. Does this mean that, say, eating ice-cream causes significant groups of children to go sugar-crazy and fall in a lake? Or even more bizarrely, that while people are drowning they suddenly consume a lot of ice-cream? Well, unsurprisingly, no. Ice-cream sales tend to go up in summer, a time when people also spend more time swimming outdoors, so rising ice-cream sales and increased drownings are both caused by warm weather but aren’t actually related directly.

If you think this is silly and that no-one would make such a basic causation-and-correlation mistake: needless to say, think again. Public Health experts in the 1940s noticed a correlation between polio cases and ice-cream consumption; they recommended cutting out ice-cream to protect against the disease. It later turned out that, you guessed it, polio outbreaks were more common in summer and ice-cream eating was more common in summer, and polio and ice-cream had nothing to do with each other.\*

Another great example is that the average price of rum is strongly correlated with the average wages of kindergarten-teachers. Do rising childcare wages give more spare cash to exasperated teachers, bidding up rum prices? Or do higher rum prices cause people to drink less, therefore work harder, therefore spend more money on their childrens’ kindergartens? Neither, obviously: the price of basically-everything rises over time due to inflation, and this extraneous factor affects kindergarten teachers’ wages and rum prices and the cost of houses and MPs’ salaries and almost-everything else. You can show correlation between any two price-rises and, if you’re a devious sort of person, ignore the fact that both increases are caused by an invisible third factor, the general fact that prices tend to rise over time. In fact, lots of trends tend to increase over time, and if you want to get really silly you can start to point out correlations between *any two* such variables, ignoring the invisible third factor (that many things increase over time, in general).

A well-known newspaper once wrote about a “striking correlation” between the number of miles driven per licensed American driver and U.S. obesity rates six years later— the hypothesis was that when Americans drove more they exercised less, and (following a time-lag to allow for the change to affect physiques) they got fat. The newspaper, although straining to note that “these predictions come with a strong caveat: correlation does not equal causation,” nonetheless seemed very impressed at the “near-perfect correlation.” A famous economist, Justin Wolfers, went one better and showed an even

better correlation between American obesity rates and his age. As Wolfers writes: “I’m not arguing that my aging is causing higher obesity. Rather, when you see a variable that follows a simple trend almost any other trending variable will fit it: miles driven, my age, the Canadian population, total deaths, food prices, cumulative rainfall, whatever.”\*

*Correlation, causation and endogeneity*

At this point, the astute reader might wonder to herself: “couldn’t all the examples above be expressed as endogeneity problems?” At which the proud author would sob loudly into his laptop, and tell everyone who would listen about how quickly Astute Reader had grown up, and wasn’t it only yesterday that she was browsing the introduction? Yes, indeed, correlation-is-not-causation is a specific example of an endogeneity problem. Take ice-cream and drownings. Our original (incorrect) formula was:

$$\mathbf{A(\text{ice-cream sales}) = drownings + error}$$

Our omitted variable was “outdoor temperature,” which we’ve lumped into our error term but which is in fact correlated with both ice-cream sales and drownings. The correct formulas would be (something like):

$$\begin{aligned} \mathbf{Y(\text{temp.})} &= \mathbf{\text{ice-cream sales} + error} \\ &\text{and} \\ \mathbf{Z(\text{temp.})} &= \mathbf{\text{drownings} + error} \end{aligned}$$

with any apparent connection between ice-cream sales and drownings actually driven by the common independent variable “outdoor temperature.” Correlation is not causation: both outcomes are endogenous to an omitted variable that has been wrongly lumped into the error term.

# CHAPTER THREE:

## BAYES

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*Bayes matters*

You come home early one day to find that your girlfriend, who told you she was going home because she wasn't feeling well, actually snuck out to dinner with her ex. (I hate to be the one to tell you this, but really, you had to know). What is the probability that she's cheating on you? More importantly: how should you *calculate* the probability that she's cheating on you? What factors do you need to take into account? You won't, and shouldn't, assign numbers here; they'd be thoroughly arbitrary ("Honey, there seems to be a 23.97% probability that you're cheating on me. Plus or minus 0.01%"). But you're going to make an implicit probabilistic assessment anyway ("She's definitely cheating on me;" "I think she might be cheating on me;" "I'm only slightly worried that she's cheating on me;" etc. etc.), so you might as well make the assessment good.

The question here is something called a *conditional probability*, that is, the probability of X given Y — here, the probability that she's cheating *given* that she snuck off to dine with her ex. So long as two things are in some way related, the conditional probability for X given Y will not be the same as the simple probability of X all on its own. For instance, the simple probability that a randomly-chosen burrito is going to be tasty might be 0.4. The conditional probability that it's going to be tasty *given* that your friend recommended this particular restaurant might be 0.8 — her recommendation is a non-random indicator of burrito quality. And the conditional probability that the burrito will be tasty *given* that your friend recommended the place, *and* that you've previously eaten quesadillas here and they were excellent, *and* that it won some prestigious burrito-making award, might be 0.99. Statisticians use the symbol  $P(X|Y)$  to represent the probability of X happening given that Y did; the vertical bar represents the phrase "given that."

Now to return to our girlfriend-problem. We have a hypothesis about the world (the hypothesis that she might be cheating on us) and a new piece of evidence (she snuck off to dinner without telling us). The Reverend Thomas Bayes was an 18th Century British clergyman who figured out exactly how to deal with these kinds of problems, where we want to know the probability that our hypothesis (the *why*, is, our causal explanation for *why* something happened) is correct given a new piece of evidence (the *what*, is, something that has happened). Bayes' insight was that the conditional probability  $P(\text{Hypothesis}|\text{Evidence})$  depends on four different things— well, they're kind of connected so there aren't technically four of them, but let's call it four different things for now.

First,  $P(\text{Hypothesis}|\text{Evidence})$  depends on the probability  $P(\text{Evidence}|\text{Hypothesis})$ : if a piece of evidence was extremely probable under a given hypothesis, the existence of the evidence makes the hypothesis more probable that the hypothesis was correct. This makes sense: if our hypothesis strongly predicted that a particular piece of evidence would show up, and that piece of evidence does indeed show up, then it is a little more probable that our hypothesis was correct (because if our hypothesis had predicted a particular piece of evidence, and that evidence had *not* shown up, then we'd be forced to conclude that our hypothesis was probably wrong).

Second, it depends on the probability that the hypothesis was correct *before* we saw the new evidence; statisticians call this the "prior probability." New evidence can be used to help "update" our beliefs — that is, after seeing a new piece of evidence, our previous hypothesis can become more

less probable. But we must never forget the prior probability that we started with. If the star striker of your favourite soccer team gets injured—new evidence—then it becomes less probable than it has been that your team will win the next match. However, the *absolute* probability that they will win the next match is still very dependent on how good they are, and how good the other team is, and how probable their win was before the injury came along. The new evidence (the striker's injury) can only cause us to update our prior beliefs; it doesn't act in a vacuum, and couldn't possibly tell us how probable a win is in the absence of prior beliefs.

The third and fourth factors for the Bayesian analysis are mirrors of the first two, but with regard to the *alternative hypotheses* that compete with our own one to explain the world around us. The alternative hypotheses are all the other possible ways to explain what happened; all the things *apart from* our hypothesis that could have caused the evidence to arise. For example, perhaps you go to your neighbour's house and the family dog jumps up and licks you. "The dog is licking me!" you exclaim. "This new evidence supports my hypothesis that the dog really likes me." But as a good statistician you must also consider alternative hypotheses: for example, that the dog licks everyone, or that your aftershave smells like meat. These alternatives, and many others, could also explain the evidence you're seeing.

The third factor, then, is the probability of the new evidence given the alternative hypotheses, and the fourth factor is the prior probabilities of each of the alternative hypotheses. Essentially, the same arguments as above apply in reverse: the more probable the evidence under alternative hypotheses, and the more probable those hypotheses were to begin with, the more probable it is that the new evidence has appeared due to these alternative causes, and so the *less* probable it is that our original hypothesis is correct. Conversely, the new evidence argues much more strongly for our initial hypothesis if the piece of evidence is highly *improbable* under the reasonable alternative hypotheses, or if those alternative hypotheses were initially very improbable.

Don't worry if that all seemed confusing, or if you're not sure you absorbed what you just read. Unfortunately Bayes' Theorem is just one of those things you have to chew on for a while before it makes sense. Let's look at a more concrete example. Suppose that a note shows up on my desk purporting to reveal a secret crush from the most beautiful girl in the class. It is indeed somewhat probable that the girl would send such a message if she had a crush on me:  $P(\text{message}|\text{crush})$  is moderately-high. Depending on how attractive I am, perhaps  $P(\text{crush})$  — the probability that she really does have a crush on me — is also non-negligible. Unfortunately, there are still the alternative hypotheses to consider: for example, that my friends are playing a prank. The probability that I would receive such a message if my friends were playing a prank —  $P(\text{message}|\text{prank})$  — is fairly high, and the probability that my friends would play a prank on me,  $P(\text{prank})$ , is extremely high. This means that, overall, the existence of the message becomes weak evidence for the hypothesis that the girl has a crush on me; the alternative hypotheses explain the evidence just as well, and were more probable before the evidence showed up.

#### Formula for Bayes

Reverend Bayes was in fact the source for more than this little theorem — his work was the cornerstone for an entire statistical philosophy commonly known as (this is going to shock you) Bayesian Statistics. There is an enormous amount of controversy in the statistics world about whether Bayesianism is really the correct way to think about probabilities, as opposed to another school of thought called Frequentism, and if you ever find yourself at a statistics conference you must promise (promise!) not to mention that this book flagrantly side-stepped the Bayesian-Frequentist debate —

this is one of those questions that inspires an enormous amount of passion and argument from the people who really care about it. For our purposes, though, the philosophical debate is not critical. Bayes' Theorem is a great tool for thinking about probabilities, regardless of deeper questions about the true nature of probabilities in our universe. Bayes' Theorem can be expressed as a simple formula. Take a deep breath, it's honestly (honestly!) not as bad as it looks:

$$P(\text{Hypothesis}_1|\text{Evidence}) = \frac{P(\text{Evidence}|\text{Hypothesis}_1)P(\text{Hypothesis}_1)}{P(\text{Evidence})}$$

$P(X)$  = the probability of  $X$  occurring,  
 $P(X|Y)$  = the probability of  $X$  occurring **given** that  $Y$  occurred.

What does that mean? Well, let's return to our example with the note from the beautiful girl (I liked that example). Our Hypothesis<sub>1</sub> was that I received the note because the girl had a crush on me and the probability we're trying to assess is  $P(H_1|E)$ : the probability that she really does have a crush on me given that I received the note.  $P(E|H_1)$  was the probability that she would send the note if she had the crush;  $P(H_1)$  was the probability that she had a crush on me before we knew the note existed.  $P(E)$  is the total probability of seeing the evidence under all possible hypotheses; we get it by adding together  $P(E|H_1)P(H_1)$  and  $P(E|H_2)P(H_2)$  and  $P(E|H_3)P(H_3)$ , etc. etc., until we have gone through all the reasonable alternative hypotheses: the total probability of seeing the evidence is a function of how probable the evidence was in each possible state of the world, weighted by how probable that state of the world was to begin with, so we arrive at  $P(E)$  by summing  $P(E|H)P(H)$  for all possible  $H$ 's. It's crucial that we include our own hypothesis,  $H_1$ , in the calculation of  $P(E)$  — after all, our hypothesis is one of the relevant states of the world under which the new evidence might occur.

#### *Assimilating evidence*

Now that we know the formula, we can start to explore its consequences. We know that there are four elements to consider when thinking about a conditional probability: the probability of  $E$  given  $H_1$ ; the prior probability for  $H_1$ ; the probability of  $E$  given  $H_{\text{others}}$ ; and the prior probabilities for  $H_{\text{others}}$ . As previously discussed, if  $P(E|H_1)$  rises then  $P(H_1|E)$  also rises; this makes sense, because the more probable the new evidence under our hypothesis, the more that seeing the new evidence makes our hypothesis more probable itself. If the prior probability  $P(H_1)$  rises then the posterior probability  $P(H_1|E)$  also rises; this makes sense, because the higher the hypothesis' probability was to begin with, the higher it will be after any given piece of new evidence is introduced. Conversely, if  $P(E|H_{\text{others}})$  rises then the posterior probability  $P(H_1|E)$  falls; this makes sense, because the more probable the new evidence is under the alternative hypotheses, the less the existence of the new evidence makes us certain that our Hypothesis<sub>1</sub> was correct. Finally, if the posterior probability  $P(H_{\text{others}})$  rises then the posterior probability  $P(H_1|E)$  falls; this makes sense, because the more probable the alternative hypotheses were to begin with, the more probable they will be after our new evidence arrives, and the less support the new evidence provides for our own original hypothesis — the new evidence might be proof, instead, that one of the *other* hypotheses was correct.



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