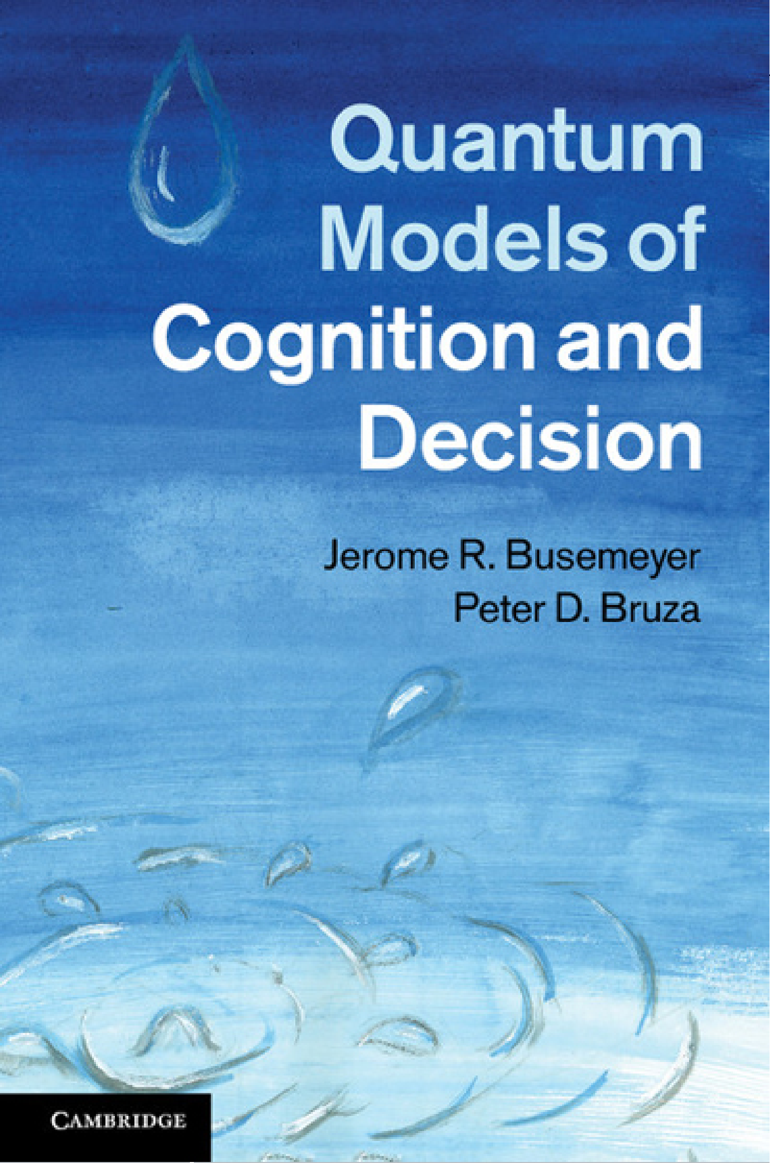
Quantum Models of Cognition and Decision
Jerome R. Busemeyer
Peter D. Bruza
CAMBRIDGE

Quantum Models of Cognition and Decision

Much of our understanding of human thinking is based on probabilistic models. This innovative book by Jerome R. Busemeyer and Peter D. Bruza argues that, actually, the underlying mathematical structures from quantum theory provide a much better account of human thinking than traditional models. They introduce the foundations for modelling probabilistic-dynamic systems using two aspects of quantum theory. The first, "contextuality," is a way to understand interference effects found with inferences and decisions under conditions of uncertainty. The second, "quantum entanglement," allows cognitive phenomena to be modelled in non-reductionist ways. Employing these principles drawn from quantum theory allows us to view human cognition and decision in a totally new light. Introducing the basic principles in an easy-to-follow way, this book does not assume a physics background or a quantum brain and comes complete with a tutorial and fully worked-out applications in important areas of cognition and decision.

Jerome R. Busemeyer is a Professor in the Department of Psychological and Brain Sciences at Indiana University, Bloomington, USA. His research includes mathematical models of learning and decision making, and he has formulated a dynamic theory of human decision making called decision field theory. Professor Busemeyer has published over 100 articles in cognitive and decision science journals, including *Psychological Review*, and was Chief Editor of the *Journal of Mathematical Psychology* from 2005 to 2010.

Peter D. Bruza is a Professor in the Faculty of Science and Technology at Queensland University of Technology, Brisbane, Australia. His research intersects information retrieval, cognitive science, and applied logic. He is a pioneer and co-instigator of the field of quantum interaction (QI) and serves on the steering committee of the quantum interaction symposia. Professor Bruza also serves on the editorial boards of *Information Retrieval*, *Journal of Applied Logic*, *The Logic Journal of the IGPL*, and the "Information Science and Knowledge Management" book series.

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《周易》有云：穷则变、变则通、通则久。

⟨⟨Yi Jing⟩⟩ Book states: ANY circumstance hitting a limit will begin to change.
Change will in turn lead to an unimpeded state, and then lead to continuity.

This book is dedicated to the person who inspired this amazing journey,
the first author's wife.

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Preface

Rationale

The purpose of this book is to introduce the application of quantum theory to cognitive and decision scientists. At first sight it may seem bizarre, even ridiculous, to draw a connection between cognition and decision making – research lying within the realm of day-to-day human behavior – on the one hand and quantum mechanics – a highly successful theory for modelling subatomic phenomena – on the other hand. Yet there are good scientific reasons for doing so, which is leading a growing number of researchers to examine quantum theory as a way to understand perplexing findings and stubborn problems within their own fields. Hence this book. Given the nascent state of this field, some words of justification are warranted. The research just mentioned is not concerned with modelling the brain using quantum mechanics, nor is it directly concerned with the idea of the brain as a quantum computer. Instead it turns to quantum theory as a fresh conceptual framework for explaining empirical puzzles, as well as a rich new source of alternative formal tools. To convey the idea that researchers in this area are not doing quantum mechanics, various modifiers have been proposed to describe this work, such as quantum-like models of cognition, cognitive models based on quantum structure, or generalized quantum models.

There are two aspects of quantum theory which open the door to addressing problems facing cognition and decision in a totally new light. The first is known as "contextuality" of judgments and decisions, which is captured in quantum theory by the idea of "interference": the context generated by making a first judgment or decision interferes with subsequent judgments or decisions to produce order effects, so that judgments and decisions are non-commutative. The second aspect relates to "quantum entanglement." Entanglement is a phenomenon whereby making an observation on one part of the system instantaneously affects the state in another part of the system, even if the respective systems are separated by space-like distances. The crucial point about entanglement relevant to this book is that entangled systems cannot be validly decomposed and modelled as separate subsystems. This opens the door to developing quantum-like models of cognitive phenomena which are not decompositional in nature. For example, the semantics of concept combinations would seem to be non-compositional, and quantum theory provides formal tools to model these as non-decomposable interacting systems. Similar applications appear in human memory. Most models consider words as separate entities – new models are made possible by going beyond this assumption and, for example, modelling a network of word associates as a non-decomposable system.

It is important to note the authors are agnostic toward the so-called “quantum mind” hypothesis, which assumes there are quantum processes going on in the brain. We motivate the use of quantum models as innovative abstractions of existing problems. That is all. These abstractions have the character of idealizations in the sense there is no claim as to the validity of the idealization “on the ground.” For example, modelling concept combinations as quantum entangled particles involves no claim as to whether there is associated physical entanglement going on somewhere in the brain. This may seem like an easy way out, but is not that different than idealizations employed in other areas of science. For example, in neural dynamical models of the brain, continuous state and time differential equations are used to model growth of neural activation, even though actually there are only a finite number of neurons and each one only fires in an all or none manner. In short, we apply mathematical structures from quantum mechanics to cognition and decision without attaching the physical meaning attributed to them when applied to the human behavioral phenomena. In fact, many areas of inquiry that were historically part of physics are now considered part of mathematics, including complexity theory, geometry, and stochastic processes. Originally they were applied to physical entities and events. For geometry, this was shapes of objects in space. For stochastic processes, this was statistical mechanics of particles. Over time they became generalized and applied in other domains. Thus, what happens here with quantum mechanics mirrors the history of many, if not most, branches of mathematics.

The cognitive revolution that occurred in the 1960s was based on classical computational logic, and the connectionist/neural network movements of the 1970s were based on classical dynamic systems. These classical assumptions remain at the heart of both cognitive architecture and neural network theories, and they are so commonly and widely applied that we take them for granted and presume them to be obviously true. What are these critical but hidden assumptions upon which all traditional theories rely? Quantum theory provides a fundamentally different approach to logic, reasoning, probabilistic inference, and dynamic systems. For example, quantum logic does not follow the distributive axiom of Boolean logic; quantum probabilities do not obey the law of total probability; quantum reasoning does not obey the principle of monotonic reasoning; and quantum dynamics can evolve along several trajectories in parallel rather than be slave to a single trajectory as in classical dynamics. Nevertheless, human behavior itself does not obey all of these restrictions. This book will provide an exposition of the basic assumptions of classic versus quantum models of cognition and decision theories. These basic assumptions will be examined, side by side, in a parallel and elementary manner. For example, classical systems assume that measurement merely observes a preexisting property of a system; in contrast, quantum systems assume that measurement actively creates the existence of a property in a system. The logic and mathematical foundation of classic and quantum theory will be laid out in a simple and elementary manner that uncovers the mysteries of both theories. Classic theory will emerge to be seen as a possibly overly restrictive case of the more general quantum theory. The fundamental implications of these contrasting assumptions will be examined closely with concrete examples and applications to cognition and decision making. New research programs in cognition and decision making, based on quantum theory, will be reviewed.

Book chapters

Chapter 1 provides the motivation for why one might be interested in applying quantum theory to cognition and decision making. In this chapter, we give a quick glance at several applications, including perception, conceptual judgments, decision making, and information retrieval. Also, this chapter briefly reviews some of the previous history and connections made between psychology and quantum physics and places the current ideas within this larger framework of research. Chapter 2 provides a simple and intuitive introduction to the basic axioms of quantum probability theory, alongside a comparison with the basic axioms of classic probability theory, and we also provide a clear *psychological* interpretation of the quantum axioms. The chapter includes simple numerical examples, calculations, and simple computer programs that provide clear and concrete ideas about how to use quantum theory to compute probabilities for cognitive and decision-making applications. Only linear algebra is needed for this introduction, which will be introduced and explained in a simple tutorial manner. No physics background is required. The next five chapters describe applications of the theory presented in Chapter 2. This includes applications to order effects on attitude judgments in Chapter 3, explanations for human probability judgment errors in Chapter 4, quantum models of conceptual combination judgments in Chapter 5, a detailed application of a quantum model to the conjoint memory recognition paradigm in Chapter 6, and a quantum model of the human mental lexicon in Chapter 7. Chapter 8 introduces the dynamic principles of quantum theory in a simple step-by-step manner with numerical examples and simple-to-use computer programs. This chapter also identifies fundamental differences between simple classical dynamic systems and quantum dynamic systems by presenting a parallel development of classic Markov and non-classic quantum processes. Chapter 9 applies the dynamic principles of the previous chapter to several paradoxical findings of decision making that cannot be easily explained by traditional decision models, including Markov models. Chapter 10 introduces some basic concepts of quantum computing and contrasts these ideas with production rule systems, connectionist networks, fuzzy set theory, and Bayesian inference theory. Computer code for analyzing various logic inference problems under uncertainty using quantum computing are provided. Chapter 11 introduces the problem of learning with quantum systems and reviews work on quantum neural networks. Finally, Chapter 12 summarizes the progress made toward applying quantum theory to cognitive and decision sciences thus far, and provides a view of future possibilities. This chapter also includes a debate with a skeptic (actually previous reviewers) about the advantages and disadvantages of using a quantum approach to cognition and decision making, as well as different ways to understand the biological basis of quantum computations by the brain. An appendix is included to review some additional mathematics needed for understanding and using more advance parts of quantum theory, and to present technical proofs that are too long to be included in the main text.

In our experience thus far, people either love or hate these ideas, but no one remains unaffected. We challenge you to make your own opinion.

Jerome R. Busemeyer, Indiana University, USA
Peter Bruza, Queensland University of Technology, Australia

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Why use quantum theory for cognition and decision? Some compelling reasons

Why should you be interested in quantum theory applied to cognition and decision? Perhaps you are a physicist who is curious whether or not quantum principles can be applied outside of physics. In fact, that is one purpose of this book. Perhaps you are a cognitive scientist who is interested in representing concepts by vectors in a multidimensional feature space. This is essentially the way quantum theory works too. Perhaps you are a decision scientist who is trying to understand how people make decisions under uncertainty. Quantum theory could provide some interesting new answers. Generally speaking, *quantum theory is a new theory for constructing probabilistic and dynamic systems*, and in this book we apply this new theory to topics in cognition and decision. Later in this chapter we will give some specific examples, but let us step back at this point and try to understand the more general principles that support a quantum approach to cognition and decision.

1.1 Six reasons for a quantum approach to cognition and decision

Quantum physics is arguably the most successful scientific theoretical achievement that humans have ever created. It was created to explain puzzling findings that were impossible to understand using the older classical physical theory, and it achieved this by introducing an entirely new set of revolutionary principles. The older classical physical theory is now seen as a special case of the more general quantum theory. In the process of creating quantum mechanics, physicists also created a new theory of probabilistic and dynamic systems that is more general than the previous classic theory (Pitowski, 1989). This book is not about quantum physics per se, but instead it explores the application of the probabilistic dynamic system created by quantum theory to a new domain – the field of cognition and decision making. Almost all previous modelling in cognitive and decision sciences has relied on principles derived from classical probabilistic dynamic systems. But these fields have also encountered puzzling findings that also seem impossible to understand within this limited framework.

Quantum principles may provide some solutions. Let us examine these principles to see why they may be applicable to the fields of cognition and decision.

1.1.1 Judgments are based on indefinite states

According to many formal models (computational or mathematical) commonly used in cognitive and decision sciences (such as Bayesian networks, or production rules, or connectionist networks), the cognitive system changes from moment to moment, but at any specific moment it is in a definite state with respect to some judgment to be made. To make this clearer, let us take a simple example. Suppose you are a member of a jury and you have just heard conflicting evidence from the prosecutor and defense. Your job is to weigh this evidence and come up with a verdict of guilty or not. Suppose your subjective probability of guilt is expressed on a $p \in [0,1]$ probability scale. Formal cognitive models assume that at each moment you are in a definite state with respect to guilt – say a state that selects a value p such that $p > 0.50$ or a state that produces p such that $p \leq 0.50$ (in other words, p is a function of the current state of the system). Of course, the model does not know what your true state is at each moment, and so the model can only assign a probability to you responding with $p > 0.50$ at that moment. But the model is stochastic only because it does not know exactly what trajectory (definite state at each time point) you are following. A stochastic model postulates a sample space of trajectories, along with a measure that assigns probabilities to sets of trajectories. But according to a stochastic model, once a trajectory is sampled (e.g., once a seed is selected to start a computer simulation), then the system deterministically jumps from one definite state (e.g., respond with $p > 0.50$) to another (e.g., respond with $p \leq 0.50$) or stays put across time. The states are pointwise and dispersion free and probabilities only arise from sampling different trajectories across new replications (e.g., starting the computer simulation over again with a new seed). In this sense, cognitive and decision sciences currently model the cognitive system as if it was a *particle* producing a definite sample path through a state space.

Quantum theory works differently by allowing you to be in an *indefinite* state (formally called a *superposition* state) at each moment in time before a decision is made. Strictly speaking, being in an indefinite or superposition state means that the model *cannot* assume either (a) you are definitely in a guilty state (e.g., a state that responds with $p > 0.50$) or (b) you are definitely in a not guilty state (e.g., respond with $p \leq 0.50$) at some moment. You may be in an indefinite state that allows both of these definite states to have *potential* (technically called state amplitudes) for being expressed at *each* moment (Heisenberg, 1958). (This does *not* mean you are definitely in both states simultaneously at each moment.) Intuitively, if you are in an indefinite state, then you do not necessarily think the person is guilty and at the same time you do not necessarily think the person is not guilty. Instead, you are in a superposition state that leaves you *conflicted, or ambiguous, or confused, or uncertain* about the guilty status. The potential for guilt may be greater than the potential for not guilty at one moment, and these potentials (amplitudes) may change from one moment to the next moment, but both answers are potentially available at *each* moment. In quantum theory, there is *no* single trajectory or sample path across time before making a decision, but instead there is a smearing of potentials across states that flows across time. In this sense, quantum theory allows one to model the cognitive system as if it was a *wave* moving across time over the state space until a decision is made. However, once a decision is reached, and uncertainty is resolved, the state becomes definite as if the wave collapses to a point like a particle. Thus, quantum systems require *both* wave (indefinite) and particle (definite) views of a cognitive system.

We argue that the wave nature of an indefinite state captures the psychological experience of conflict, ambiguity, confusion, and uncertainty; the particle nature of a definite state captures the psychological experience of conflict resolution, decision, and certainty.

1.1.2 Judgments create rather than record

According to many formal models, the cognitive system may be changing from moment to moment, but what we record at a particular moment reflects the state of the system as it existed immediately before we inquired about it. So, for example, formal cognitive models assume that if a person watches a disturbing scene and we ask the person a question such as "Are you afraid?", then the answer reflects the state of the person regarding that question just before we asked it. If instead we asked the person "Are you excited?" then the answer again reflects the state regarding this other question just before we asked it.

One of the more provocative lessons learned from quantum theory is that taking a measurement of a system creates rather than records a property of the system (Peres, 1998). Immediately before asking a question, a quantum system can be in an indefinite state. For example, the person may be ambiguous about his or her feelings after watching a disturbing scene. The answer we obtain from a quantum system is constructed from the interaction of the indefinite state and the question that we ask (Bohr, 1958). This interaction creates a definite state out of an indefinite state. For example, the person may have been ambiguous about their feelings after the disturbing scene, but this state becomes more definite after answering the question about being afraid. If the answer is "Yes, I feel afraid," then the person acts accordingly. This is, in fact, the basis for modern psychological theories of emotion (Schachter & Singer, 1962). Decision scientists also argue that beliefs and preferences are constructed on line rather than simply being read straight out of memory (Payne *et al.*, 1992). For example, a person may initially be in an indefinite state about a set of paintings on display, but if the person is asked to choose one as a gift, then a preference order is constructed on line for the purpose.

We do not wish to argue that every answer to every question involves the construction of an opinion. For many questions you do have a stored answer that is simply retrieved on demand (e.g., Have you ever read a certain book?). But other questions are new and more complex and you have to construct an answer from your current state and context (e.g., Did you like the moral theme of that book?). So we argue that the quantum principle of constructing a reality from an interaction between the person's indefinite state and the question being asked actually matches psychological intuition better for complex judgments than the assumption that the answer simply reflects a preexisting state.

1.1.3 Judgments disturb each other, introducing uncertainty

According to quantum theory, if one starts out in an indefinite state, and is asked a question, then the answer to this question will change the state from an indefinite state to one that is more definite with respect to the question that was asked. But this change in state after the first question then causes one to respond differently to subsequent questions so that the order of questioning becomes important. Consider the following popular example from social psychology. Suppose a teenage boy is directly asked "How happy are you?" the typical answer is "Everything is great." However, if this teenager is first asked "When was the last time you had a date?" then the answer tends to be "Seems like a long time ago." Following this sobering answer, a later question about happiness tends to produce a second answer that is not so sunny and rosy. Thus, the first question sets up a context that changes the answer to the next question. Consequently, we cannot define a joint probability of answers to question A and question B, and instead we can only assign a probability to the sequence of answers to question A followed by question B. In quantum physics, if A and B are two measurements and the probabilities of the outcomes depend on the order measurement, then the two measurements are non-commutative. In physics, for example, measurements of position and momentum along the same direction are non-commutative, but measurements of positions along the horizontal and vertical coordinates are commutative. Many of the mathematical properties of quantum theory arise from developing a probabilistic model for non-commutative measurements, including Heisenberg's (1927) famous uncertainty principle (Heisenberg, 1958).

Order effects are also responsible for introducing uncertainty into a person's judgments. If the first question A produces an answer that creates a definite state with respect to that question, the state created by A may be indefinite with respect to a different question B. Consider the following consumer choice example. Suppose a man is considering the purchase of a new car and two different brands are in contention: a BMW versus a Cadillac. If he directly asks himself what he prefers, he definitely answers with the BMW. But if he first asks himself what his wife prefers (she definitely wants the Cadillac) and subsequently asks himself what he prefers (after taking on his wife's perspective), then he becomes uncertain about his own preference. In this example, the question about his wife's preference disturbs and creates uncertainty about his own preference. Thus, it may be *impossible* to be in a definite state with respect to two different questions, because a definite state (technically speaking an eigenstate) for one is an indefinite state (superposition) for another. In this case, the questions are said to be *incompatible* and the incompatibility of questions is mathematically implemented by the non-commutativity of quantum measurements. Question order effects are a major concern for attitude researchers, who seek a theoretical understanding of these effects similar to that achieved in quantum theory (Feldman & Lynch, 1988).

1.1.4 Judgments do not always obey classic logic

The classic probability theory used in current cognitive and decision models is derived from the Kolmogorov axioms (Kolmogorov, 1933/1950). These axioms assign probabilities to events defined as sets. Consequently, the family of sets in the Kolmogorov theory obeys the Boolean axioms of logic. Thus, Boolean logic lies at the foundation of current probabilistic models of cognition and decision making. One important axiom of Boolean logic is the distributive axiom: if $\{G,T,F\}$ are events then $G \cap (T \cup F) = (G \cap T) \cup (G \cap F)$. Consider, for example, the concept that a boy is good (G) and the pair of concepts the boy told the truth (T) and the boy did not tell truth (falsehood, F). Suppose you are trying to decide if the boy is good but you do not know if he is truthful. According to Boolean logic, the event G can only occur in one of two ways: either $(G \cap T)$ occurs or $(G \cap F)$ exclusively. This means there are only two mutually exclusive and exhaustive ways for you to think the boy is good: he is good and truthful or he is good and he is not truthful.

From this distributive axiom, one can derive the law of total probability. Define $p(G)$ as the probability of event G, $p(T)$ is the probability of event T, $p(F)$ is the probability of event F, $p(G|T)$ is the probability of event G conditioned on knowing event T, and $p(G|F)$ is the probability of event G conditioned on knowing event F. Then the law of total probability follows from

$$\begin{aligned} p(G) &= p\left((G \cap T) \cup (G \cap F)\right) = p\left(G \cap T\right) + p(G \cap F)) \\ &= p(G)p(G|T) + p(F)p(G|F). \end{aligned}$$

This law provides the foundation for inferences with Bayes nets. The law of total probability is violated by the results of the disjunction experiment and the category – decision-making experiment in psychology and the two-slit type of experiments in physics, all of which we describe later in this chapter.

Quantum probability theory is derived from the von Neumann axioms (von Neumann, 1932/1955). These axioms assign probabilities to events defined as subspaces of a vector space (more on this in Chapter 2). The definite states form the basis for the vector space, and an indefinite or superposition state can be any point within this vector space. An important consequence of using subspaces is that the logic of subspaces does not obey the distributive axiom of Boolean logic (Hughes, 1989). For example, according to quantum logic, when you try to decide whether a boy is good without knowing if he is truthful or not, you are *not* forced to have only two thoughts: he is good and he is truthful or he is good and he is not truthful. You can have other ambiguous thoughts represented by a superposition over the truthful or not truthful attributes.

The fact that quantum logic does not always obey the distributive axiom implies that the quantum model does not always obey the law of total probability (Khrennikov, 2010). This is why the quantum model can explain the results of the disjunction experiment in psychology and the two-slit experiment in physics. Thus, quantum logic is a generalization of classic logic and quantum probability is a generalized probability theory. We argue that classic logic and classic probability theory are too restrictive to explain human judgments and decisions.

1.1.5 Judgments do not obey the principle of unicity

The classic (Kolmogorov) probability theory, which is used in current cognitive and decision models, is based on the principle of *unicity* (Griffiths, 2003). A single sample space is proposed which provides a complete and exhaustive description of all events that can happen in an experiment.[1] This follows from the Boolean algebra used in classic theory: if A is an event and B is another event from an experiment, then $A \cap B$ must be an event too, and repeated application of this principle leads to intersections that cannot be broken down any further (the atoms or elements or points of the sample space). All events can be described by unions of the atoms or elements or points of the sample space. If you think about this for a while, this is a tremendous constraint on a theory. We argue that it is oversimplifying the extremely complex nature of our world.

Let us examine the consequence of assuming unicity for experiments on human probability judgments. Suppose we do an experiment in which we ask a person to describe the likelihood of various future events with respect to future political history. Perhaps a person has the knowledge to do this within a single sample space. But then we can also ask the same person to describe the likelihood of future events with respect to progress in science. Now it becomes quite a stretch to imagine that the person is able to assign joint probabilities to all historical and scientific events. Instead, the person might need to fall back on one description of events (one sample space) for political futures, but use a different description of events (another sample space) for future scientific progress. To go even further, we could ask about the likelihood of events concerning the romantic and marital relations of Hollywood movie stars. Surely we have passed the capacity of the person who would have little or no idea about how to combine all three of these topics into a unified sample space that assigns joint probabilities to all three kinds of events.[2]

Quantum probability does not assume the principle of unicity (Griffiths, 2003). This assumption is broken as soon as we allow incompatible questions into the theory which cause measurements to be non-commutative (Primas, 2007).

[1] Kolmogorov realized that different sample spaces are needed for different experiments, but his theory does not provide a coherent principle for relating these separate experiments. This is exactly what quantum probability theory is designed to do.

[2] One could try to assume independence between questions about history, science, and Hollywood movie stars. But independence is also an overly severe restriction to impose on human judgments.