Plate Buckling in Bridges and other Structures

Björn Åkesson

Plate Buckling in Bridges and other Structures

Plate Buckling in Bridges and other Structures

Björn Åkesson

Consulting Engineer, Fagersta, Sweden

Taylor & Francis
Taylor & Francis Group
LONDON / LEIDEN / NEW YORK / PHILADELPHIA / SINGAPORE

Taylor & Francis is an imprint of the Taylor & Francis Group, an informa business

This edition published in the Taylor & Francis e-Library, 2007.

"To purchase your own copy of this or any of Taylor & Francis or Routledge's collection of thousands of eBooks please go to www.eBookstore.tandf.co.uk."

Published by: Taylor & Francis/Balkema
P.O. Box 447, 2300 AK Leiden, The Netherlands
e-mail: Pub.NL@tandf.co.uk
www.balkema.nl, www.taylorandfrancis.co.uk,
www.crcpress.com

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloging in Publication Data
Åkesson, B. (Bjorn)
Plate buckling in bridges and other structures / B. Åkesson.
p. cm.
Includes bibliographical references.
ISBN 978-0-415-43195-8 (hardcover : alk. paper)
1. Buckling (Mechanics) 2. Structural stability. I. Title.

TA656.2.A33 2007
624.1'76--dc22
2006102580

ISBN 0-203-94630-8 Master e-book ISBN

ISBN13 978-0-415-43195-8 (Hbk)
ISBN13 978-0-203-94630-5 (eBook)

Contents

Preface

As a lecturer at Chalmers University of Technology in Gothenburg, Sweden, during the years 1994–2004, I recognized the need for the students to have a pedagogical textbook concerning buckling of thin-walled plates. The books we used were often too theoretical – theory is essential, but it should be combined with practical issues as well. I therefore devoted my last two years at Chalmers to writing a textbook that would meet the needs of the students, and by extension, practising engineers. In writing the book, and delivering the information and disclosing the inner core of a complex subject, I tried to have in mind the learning process of the students.

Some may perhaps wonder – especially those readers looking for a book focusing exclusively on plane plates – why there is a chapter devoted to the buckling of shells? This final theoretical chapter ties together with the rest of the book, as there are important differences (and similarities) in the action of a shell in relation to a plane plate, which helps the reader to understand both the former and the latter. And one must also remember that even though Robert Stephenson's Britannia Bridge was built using only plane plates back in the 1850s, Stephenson, prior to the completion of the bridge, carried out tests on circular and elliptical girder tubes – one of the earliest examples of comparative tests to see the difference in buckling behaviour between different girder shapes.

January 2007
Björn Åkesson

Acknowledgement

During the ten-year-period in between 1994 and 2004 I was a lecturer and researcher within the fields of Structural Engineering and Bridge Engineering at Chalmers University of Technology in Göteborg, Sweden. My focus as a researcher was in the beginning concentrated on the fatigue life of riveted railway bridges in steel, and this was also reflected in the lectures I gave. However, the need and interest of the students did turn this focus more and more towards bridge engineering in general, and buckling of thin-walled plated bridge girders in particular. The one and only person that really did open my eyes and inspired me to gain deep knowledge within this field was Prof. Em. Bo Edlund. He has since the early 1970s been one of the leading researchers in the world concerning buckling problems of both thin-walled plated structures and cylinders. It has been a great privilege and honour working close to this extraordinarily talented man. Another good friend of mine, as well as research colleague, is Associate Prof. Mohammad Al-Emrani, with whom I spent numerous hours discussing different problems, mostly concerning fatigue, but also about buckling. Mohammad and Bo have been a great source of support during my years at Chalmers. I will also take the opportunity to thank Robert Kliger, the present professor at the department. I owe a lot to Robert, as it is entirely him, and no one else, who made it possible for me to write a textbook about buckling (an early, but short version of this book), during my last months at the department. Another good friend of mine, Jan Sandgren, has also contributed in the making of this book. He has over the years provided me with many ideas, articles and illustrations.

Björn Åkesson

List of symbols

α	reduction factor (production method, tolerance level)
β_A	the ratio between effective and gross area
γ_G	partial factor (load effect)
γ_{M0}	partial factor (resistance of cross-sections)
γ_{M1}	partial factor (resistance of members to instability)
η	knock-down factor
η	factor, taking the steel grade into account
$\overline{\eta}_1$	the ratio between design bending moment and design plastic resistance moment
$\overline{\eta}_3$	the ratio between design shear force and shear buckling design resistance
θ	slope of the panel diagonal
λ	slenderness parameter
$\overline{\lambda}_F$	slenderness parameter (concentrated load)
$\overline{\lambda}_p$	slenderness parameter (plate buckling)
λ_s	slenderness parameter (shell buckling)
$\overline{\lambda}_w$	slenderness parameter (shear buckling)
ν	poisson's ratio
ρ	reduction factor
σ_{bb}	strength of the tension field
σ_{cr}	critical buckling stress
σ_{cr}^r	critical buckling stress (reduced value)
σ_{Euler}	critical buckling stress according to the Euler theory
$\sigma_{f \cdot Ed}$	longitudinal stress in the flange
τ_{cr}	critical shear buckling stress
τ_r	shear load range (repeated loading)
φ	inclination of the tension field
χ	buckling reduction factor
χ_F	reduction factor (concentrated load)
ψ	stress value
ω	out-of-plane deflection
ω	axial deformation
ω_r	out-of-plane deflection (repeated loading)

ω_r	initial imperfection
ω_s	reduction factor (shell buckling)
A	cross-sectional area
A	constant
A_{eff}	effective net area
A_{gross}	gross area
a	plate length
a	distance between vertical stiffeners
a_c	buckling length
a/b	panel aspect ratio
b	plate width
b_e	effective width
b_{eff}	effective breadth
b_f	flange width
b_w	depth of the web
b/t	slenderness ratio
c	anchorage length in the flange
c/t	slenderness ratio
$M_{c \cdot Rd}$	design bending moment resistance
M_{sd}	design bending moment
D	plate bending stiffness
D	weight
d	depth of the web
E	modulus of elasticity (Young's modulus)
F	design concentrated (transverse) load
F_{cr}	critical buckling load
F_{Ed}	design transverse force
F_{Rd}	design transverse force resistance
F_{sd}	design concentrated (transverse) load
f_{rd}	design strength resistance
f_y	yield strength
f_{yf}	yield strength of the flange
f_{yw}	yield strength of the web
G	weight
g	weight per meter
g	width of the tension field
h	depth of the web
h_w	depth of the web
I	second moment of area
I_{st}	second moment of area (stiffener)
i	radius of gyration
k	buckling coefficient
k_F	buckling coefficient (concentrated load)
k_τ	buckling coefficient (shear load)
L	span length

L_{eff} effective length
L_{cr} buckling length
l_c buckling length
l_r measure length (initial imperfection)
l_y effective loaded length
M bending moment
$M_{c \cdot Rd}$ design bending moment resistance
M_{Ed} design bending moment
M_f flange only bending moment
$M_{f \cdot Rd}$ flange only bending moment resistance
M_{pl} plastic resistance moment
$M_{pl \cdot Rd}$ design plastic resistance moment
M_{sd} design bending moment
m number of half-sine waves (long. direction)
m_1 factor
m_2 factor
$N_{b \cdot Rd}$ design axial force resistance
$N_{b \cdot sd}$ design axial force
$N_{c \cdot sd}$ design axial force
$N_{c \cdot Rd}$ design axial force resistance
n number of half-sine waves (transv. direction)
P axial load
P_{cr} critical buckling load (axial load)
P_{max} maximum load-carrying capacity
P_{sd} design load
Q weight
q evenly distributed load
q evenly distributed load (design value)
q_{cr} critical buckling load
R anchored load
$R_{a \cdot Rd}$ design crippling resistance
$R_{y \cdot Rd}$ design crushing resistance
r/t slenderness ratio
s_c anchorage length (compression flange)
s_s stiff bearing length
s_t anchorage length (tension flange)
s_y dispersion length
t plate thickness
t_{eq} equivalent plate thickness
t_f flange thickness
t_w web plate thickness
V shear load
ΔV critical shear buckling resistance
$V_{bb \cdot Rd}$ design shear buckling resistance

$V_{bf,Rd}$	design shear buckling resistance (flange contribution)
$V_{b,Rd}$	design shear buckling resistance
$V_{bw,Rd}$	design shear buckling resistance (web contribution)
$V_{bw.Rd}$	design shear buckling resistance
V_{Ed}	design shear force
V_{sd}	design shear force
W_{eff}	elastic section modulus (effective net section)
W_{el}	elastic section modulus
W_{pl}	plastic section modulus
$y_{n.a.}$	position of the neutral axis

Chapter 1

Introduction

Buckling is an instability phenomenon that can occur if a slender and thin-walled plate – plane or curved – is subjected to axial pressure (i.e. compression). At a certain given critical load the plate will buckle very sudden in the out-of-plane transverse direction. The compressive force could besides coming from pure axial compression, also be generated by bending moment, shear or local concentrated loads, or by a combination among these. If the structural element is compact, the load-carrying capacity is governed by the yield stress of the material, rather than buckling strength capacity. If instead the element is slender and/or thin-walled, the buckling strength is governed by the so-called slenderness ratio – the buckling length over the radius of gyration for global buckling of a column or a strut, or the loaded width over the thickness of the plate for local buckling. A special form of instability, that has to be considered with great care in design, is the combined global and local buckling risk of a slender and thin-walled axially loaded plated column – the capacity could here be much lower than the two buckling effects analyzed separately. In this book, however, we will only concentrate on the latter instability phenomenon, i.e. local buckling.

Eurocode defines four cross-section classes with reference to the local buckling risk. The parameter that governs what particular class a cross-section belongs to is the slenderness ratio of the individual plates of the cross-section mentioned above. The level of the slenderness ratio then governs the ability (or inability) for plastic rotational capacity, i.e. elongation at the tension side, and compression (with possible buckling risk) at the other side, for a girder subjected to a bending moment. These four classes in the Eurocode (Class 1–4) are for girders subjected to a bending moment defined as follows below. The maximum possible loading capacity (q_{sd}) in the ultimate loading state is given by this condition (where index c is telling us that it is the ability to carry compressive stresses – with respect to the local buckling risk – that governs the maximum capacity) (Eq. 1.1):

$$M_{sd}^{max} \leq M_{c.Rd} \tag{1.1}$$

1.1 Class 1

The cross-section is so compact (read: with a sufficiently low slenderness ratio/high plastic rotational capacity) that it is possible to form a mechanism with plastic hinges in a statically in-determinate system. This gives the possibility to level out the bending moment differences in ultimate limit state design. Girders in class 1 are normally standard hot-rolled profiles (Fig. 1.1).

1.2 Class 2

The cross-section is also here compact, but not enough to be able to form a mechanism in a statically indeterminate system. The design moment distribution is for the elastic response. However, the longitudinal section with the maximum moment can be designed for full plastification over the entire cross-section height – in this case similar to class 1 cross-sections. For statically determinate systems there is no difference between the two classes. Girders in class 2 are normally also standard hot-rolled profiles (Fig. 1.2).

1.3 Class 3

The cross-section can be characterized as semi-compact, having a reduced capacity for full plastification, due to the local buckling risk on the compression side. Just as for class 2 profiles, the design moment distribution is for elastic response, however, with the difference that the maximum strained section is designed for elastic (triangular) stress distribution. Girders in class 3 are normally welded profiles (Fig. 1.3).

For unsymmetrical cross-sections (in class 3) – e.g. having a wider compression flange (than the tension flange) – yielding is accepted for the tensile stresses, however, the stresses at the compressive side limited to the yield strength at the extreme fibre. Where only the web is in class 3, and the compression flange is either in class 1 or 2, the Eurocode accepts that the properties are based on an effective class 2 cross-section, where complete yielding of the entire cross-section is accepted, with the exception of a central part of the web subjected to compression, which is neglected.

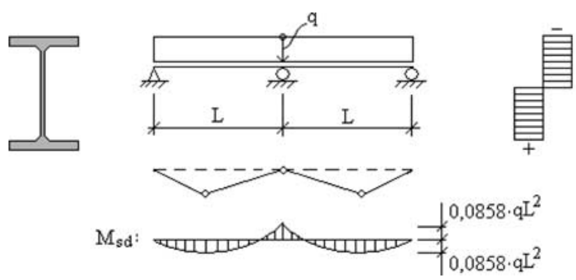

Figure 1.1 Cross-section class 1.

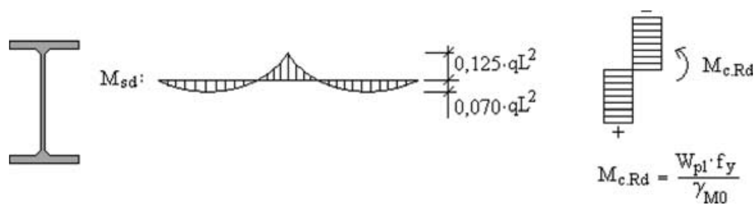

Figure 1.2 Cross-section class 2.

1.4 Class 4

The cross-section is thin-walled, i.e. having such a high slenderness ratio that buckling will occur before yielding is reached in the outermost fibre. Post-critical reserve effects enables though for yielding to be reached in the extreme fibre in the ultimate limit state design. An effective net cross-section is analyzed, where the buckled zone is removed from the gross cross-section (due to the loss of stiffness in that area). Examples of profiles in class 4 are welded bridge girders (Fig. 1.4).

For columns and struts, subjected to pure axial compression, the load-carrying capacity for profiles in class 1–3 is only reduced with respect to global (Euler) buckling risk. It is first at profiles in class 4 that the load-carrying capacity also has to be reduced for the local (plate) buckling risk.

In bridge construction, as well as in aircraft and shipbuilding industry, it is an absolute necessity to save material, and therefore the structural elements are made thin-walled and slender. To choose a compact profile (which is able to fully plastify before any local buckling risk) is not economical, as it wastes material – the increase in load-carrying capacity (in comparison to a thin-walled cross-section) is eaten up by the relative increase in cross-sectional area (compare example 9 in this book). In addition, it is absolutely necessary to keep the self-weight down, so that a good and sufficient part of the load-carrying capacity is spared for the traffic load (read: too much part of the load-carrying capacity should not be taken by the self-weight alone). A heavy and compact section bridge is also costly with respect to the extra need of foundation and substructure dimensions. High and slender girders (with thin-walled cross-sections)

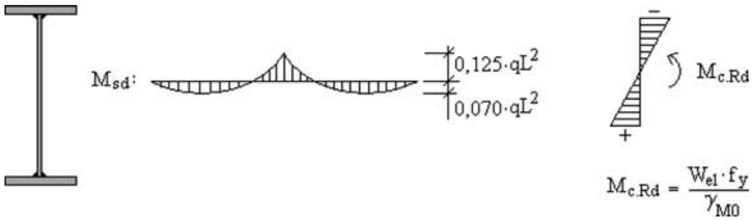

Figure 1.3 Cross-section class 3.

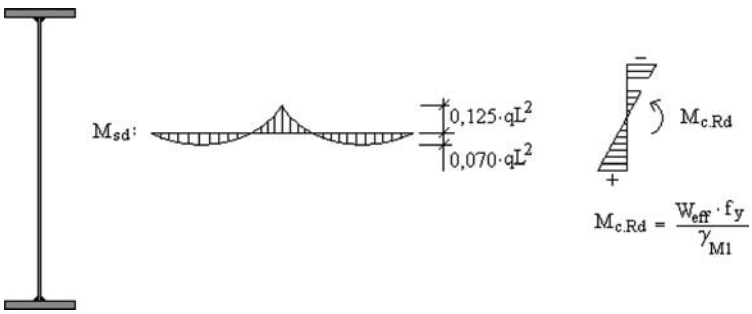

Figure 1.4 Cross-section class 4.

have also a higher stiffness relative an equivalent girder having a compact cross-section, leading to a reduced deflection under loading. In construction of buildings though, compact sections (such as standard hot-rolled profiles) are preferred, as they keep the profile depth down. These compact profiles are also more "robust", which can be needed during transport, handling and assembly.

One way of further increasing the load-carrying capacity of a slender and thin-walled plate is by the help of stiffeners, which minimize the free spacing of the parts subjected to compression. A plated bridge girder, as well as the hull of a ship, is normally stiffened in both the longitudinal and the transverse direction in order to maximize the load-carrying capacity. Provided that the stiffeners are sufficiently strong, the risk of buckling is restricted to the plate areas in between the stiffeners. The maximum load-carrying capacity of these plate panels is then governed by the plate buckling risk, however, also by taking the post-critical reserve effects into account.

The general expression for the critical buckling stress (irrespective of the type of stress distribution) is (Eq. 1.2):

$$\sigma_{cr} = k \cdot \frac{\pi^2 \cdot E}{12 \cdot (1 - \upsilon^2) \cdot \left(\dfrac{b}{t}\right)^2} \tag{1.2}$$

The so-called buckling coefficient k varies depending on the type of stress distribution, and on the quotient between the length (denoted a) and the width (denoted b) of the plate (k has its lowest value for pure axial loading in compression, which also gives the lowest value for the critical buckling stress). The quotient b/t is the slenderness (ratio) of the plate.

Plate buckling has – in contrary to global buckling of a column or a strut, or the lateral-torsional buckling of a beam – a post-critical load-carrying capacity that enables for additional loading after local buckling has occurred. A plate is in that sense inner statically indeterminate, which makes the collapse of the plate not coming when buckling occurs, but instead later, at a higher loading level. This is taken into consideration in the ultimate limit state design of plates – local buckling does not restrict the load-carrying capacity to the critical buckling stress, instead the maximum capacity consists of the two parts; the buckling load + the additional post-critical load. Global buckling of a column or a strut does not exhibit such an indeterminate behaviour, as these are statically determinate systems (having no post-critical reserve strength, i.e. no ability to redistribute load). This particular instability phenomenon – global buckling of a strut or a column – is, however, not the focus of this book.

In the coming chapter we will concentrate more in detail on the theory behind plate buckling and the load-carrying capacity of unstiffened plates in the ultimate limit state (for thin-walled cross-sections in class 4 as they are defined in Eurocode).

Chapter 2

Plate buckling theory

Consider the axially loaded "plate strut" in Fig. 2.1 (width b, length a, and thickness t), having the loaded edges supported and the unloaded edges free. The strut has the appearance of a plate, however, but not treated as such – we will instead use the classical Euler theory in our following analysis (and soon come to the theory for true plate action).

When the load reaches a certain critical value, expressed as either P_{cr} or σ_{cr}, the strut buckles and collapses (read: the lateral deflection goes to infinity) (Fig. 2.2).

For any given axial loading below this critical value, it is possible to apply an additional horizontal (transverse) force without the occurrence of buckling (the strut balances both the vertical – axial – loading and the horizontal, and will deflect back

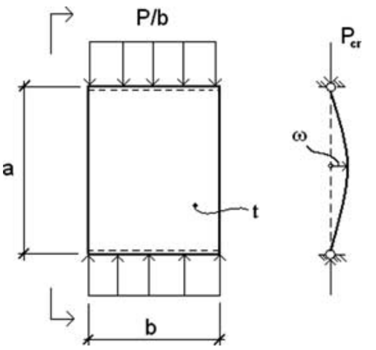

Figure 2.1 An axially loaded "plate strut".

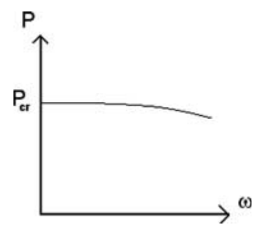

Figure 2.2 Load/displacement curve for an axially loaded "plate strut".

as soon as the horizontal load is removed). The closer the axial load is to the critical buckling load, the less the ability to carry an additional horizontal loading becomes. At exactly the critical buckling load, this ability becomes zero – the strut is then barely able to just carry the axial load.

According to the well-known Euler theory, the critical buckling load for the strut becomes (Eq. 2.1):

$$P_{cr} = \frac{\pi^2 \cdot EI}{a^2} \cdot \frac{1}{(1-\upsilon^2)} \tag{2.1}$$

This expression is adjusted with respect to the relatively large width in relation to the (buckling) length of the strut that we are studying. This adjustment is done with the quotient $1/(1-\upsilon^2)$, and this is due to the free strain deformations in the transverse direction in the centre part, in relation to the constraint at the loaded edges. In comparison to the normal appearance of a strut – where the width is small in comparison to the length – we will, for our plate strut, receive a slightly higher value for the critical buckling load due to this transverse strain divergence.

We now transform the critical buckling load (P_{cr}) given above, to an equivalent critical buckling stress (σ_{cr}), with the help of the expression for the moment of inertia of the strut (Eq. 2.2):

$$\sigma_{cr} = \frac{P_{cr}}{b \cdot t} \Rightarrow \quad \sigma_{cr} = \frac{\pi^2 \cdot E}{12 \cdot (1-\upsilon^2) \cdot \left(\frac{a}{t}\right)^2} \tag{2.2}$$

$$I = \frac{b \cdot t^3}{12}$$

Now compare the small difference between the expressions for the critical buckling stress of a plate (see Eq. 1.2) with the expression above for the (Euler) strut. If one remembers how easily this latter expression was found, then it also works as a good reminder of the former! The critical buckling stress expression for a plate has a factor k (called the buckling coefficient), and a slenderness ratio defined as the quotient b/t, instead of a/t for the strut, otherwise the expressions are similar. The last difference regarding the definition of the slenderness – that it is the width b instead of the length a over the thickness t – is very important to remember, as the length of a plate is not governing the critical buckling stress. The width b is the main parameter governing the critical buckling stress of a plate, and we will next find out – by the help of the theory behind plate buckling – why this is so.

By definition, a strut (or a column) is only supported at its loaded edges, while a plate is supported at three edges or more, and it is this fact that makes a plate have a different buckling behaviour than a strut – the transverse width b becomes the governing parameter instead of the length a. It is also in the transverse direction relative the loading direction, that plates have a capacity to develop a tension field after buckling has occurred, and by doing so – through a transverse membrane action – enable for an additional loading capacity in the so-called post-critical range.

In order to get a background to the expression for the critical buckling stress of a plate – that was given in chapter 1 (Eq. 1.2) – we start by studying the differential equation for a plate, however, not loaded in the axial direction, but in the *transverse* direction by bending (Fig. 2.3).

The differential equation gives us the relationship between the lateral out-of-plane deflection and the transverse loading q (Eq. 2.3):

$$D \cdot \left(\frac{\delta^4 \omega}{\delta x^4} + 2 \cdot \frac{\delta^4 \omega}{\delta x^2 \delta y^2} + \frac{\delta^4 \omega}{\delta y^4} \right) = q \tag{2.3}$$

where D is the plate bending stiffness (Eq. 2.4):

$$D = \frac{E \cdot t^3}{12 \cdot (1 - \upsilon^2)} \quad [\mathrm{Nm^2/m}] \tag{2.4}$$

Compare the differential equation given above (in Eq. 2.3), with the equivalent 2D-expression for a beam also subjected to bending (Fig. 2.4) (Eqs. 2.5–2.8):

$$-EI \cdot \frac{\delta^2 \omega}{\delta x^2} = M \tag{2.5}$$

$$M(x) = \frac{q \cdot L}{2} \cdot x - \frac{q \cdot x^2}{2} \tag{2.6}$$

$$M'(x) = \frac{q \cdot L}{2} - q \cdot x \tag{2.7}$$

$$M''(x) = -q \quad \Rightarrow \quad EI \cdot \frac{\delta^4 \omega}{\delta x^4} = q \tag{2.8}$$

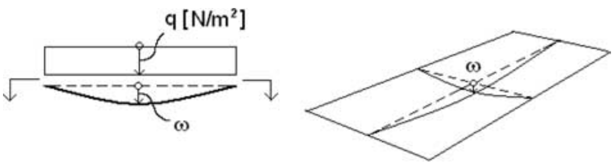

Figure 2.3 A plate loaded in the transverse direction by an evenly distributed load, *q*.

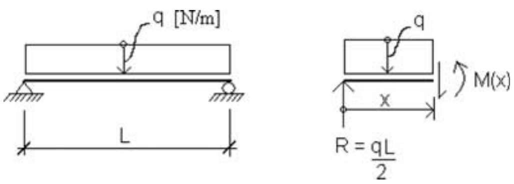

Figure 2.4 A beam subjected to bending.

Let us now continue to study a plate, however, not loaded now in bending, but instead having an in-plane axial load (Fig. 2.5).

We look for the equilibrium of a small element (having $\delta x = \delta y = 1$) in a deflected state, axially loaded with a normal force per unit width, $\sigma_x \cdot t$ [N/m] (Fig. 2.6).

This small element does not have a load in the transverse direction, i.e. $q = 0$ when we compare with the differential equation (given in Eq. 2.3). It is true that for normal forces below the critical buckling load (i.e. in the *sub-critical* range) it is required an additional transverse load/force to keep the plate in a deflected shape, however, this deflection would go back as soon as this additional load would be removed. At a certain level of the axial load ($\sigma_x = \sigma_{cr}$), the outwards going and resulting transverse force – due to the curvature – is in precise balance with the "re-bouncing" force. This exact value of the normal force (or stress) is defined as the critical value with respect to plate buckling.

The differential equation for an axially loaded plate can thus be written (Eq. 2.9):

$$D \cdot \left(\frac{\delta^4 \omega}{\delta x^4} + 2 \cdot \frac{\delta^4 \omega}{\delta x^2 \delta y^2} + \frac{\delta^4 \omega}{\delta y^4} \right) = -\sigma_{cr} \cdot t \cdot \frac{\delta^2 \omega}{\delta x^2} \tag{2.9}$$

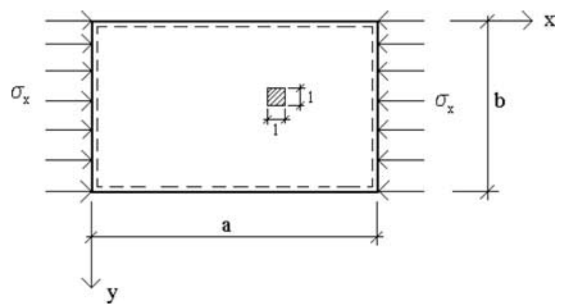

Figure 2.5 A plate loaded in the axial direction with an evenly distributed edge load.

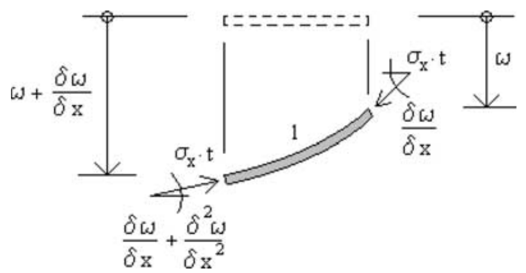

Figure 2.6 The deflected state of an axially loaded small element.

The general expression for the solution of this equation is as follows (where we have assumed a sine wave in both the longitudinal and transverse direction) (Eq. 2.10):

$$\omega = A \cdot \sin \frac{m \cdot \pi \cdot x}{a} \cdot \sin \frac{n \cdot \pi \cdot y}{b} \tag{2.10}$$

ω out-of-plane deflection
A constant
m number of half-sine waves in the longitudinal direction
n number of half-sine waves in the transverse direction
a length of the plate (unloaded edge)
b width of the plate (loaded edge)

For $n = 1$ and $m = 2$ the buckling mode looks as follows (Fig. 2.7).

We use the general expression for the solution of the out-of-plane deflection, in order to find the value of the critical buckling stress, σ_{cr} (Eqs. 2.11–2.17):

$$\frac{\delta\omega}{\delta x} = A \cdot \frac{m\pi}{a} \cdot \cos \frac{m\pi x}{a} \cdot \sin \frac{n\pi y}{b} \tag{2.11}$$

$$\frac{\delta^2\omega}{\delta x^2} = -A \cdot \frac{m^2\pi^2}{a^2} \cdot \sin \frac{m\pi x}{a} \cdot \sin \frac{n\pi y}{b} \tag{2.12}$$

$$\frac{\delta^3\omega}{\delta x^3} = -A \cdot \frac{m^3\pi^3}{a^3} \cdot \cos \frac{m\pi x}{a} \cdot \sin \frac{n\pi y}{b} \tag{2.13}$$

$$\frac{\delta^4\omega}{\delta x^4} = A \cdot \frac{m^4\pi^4}{a^4} \cdot \sin \frac{m\pi x}{a} \cdot \sin \frac{n\pi y}{b} \tag{2.14}$$

$$\frac{\delta^3\omega}{\delta x^2 \delta y} = -A \cdot \frac{m^2\pi^2}{a^2} \cdot \sin \frac{m\pi x}{a} \cdot \frac{n\pi}{b} \cdot \cos \frac{n\pi y}{b} \tag{2.15}$$

$$\frac{\delta^4\omega}{\delta x^2 \delta y^2} = A \cdot \frac{m^2\pi^2}{a^2} \cdot \sin \frac{m\pi x}{a} \cdot \frac{n^2\pi^2}{b^2} \cdot \sin \frac{n\pi y}{b} \tag{2.16}$$

$$\frac{\delta^4\omega}{\delta y^4} = A \cdot \sin \frac{m\pi x}{a} \cdot \frac{n^4\pi^4}{b^4} \cdot \sin \frac{n\pi y}{b} \tag{2.17}$$

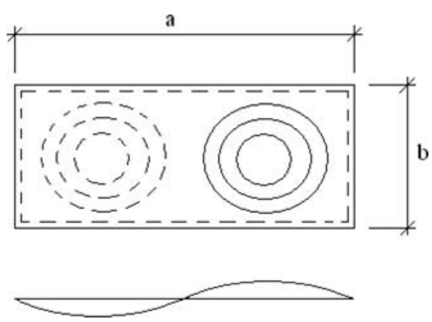

Figure 2.7 The buckling mode of a rectangular plate, having the relationship $a/b = 2$.