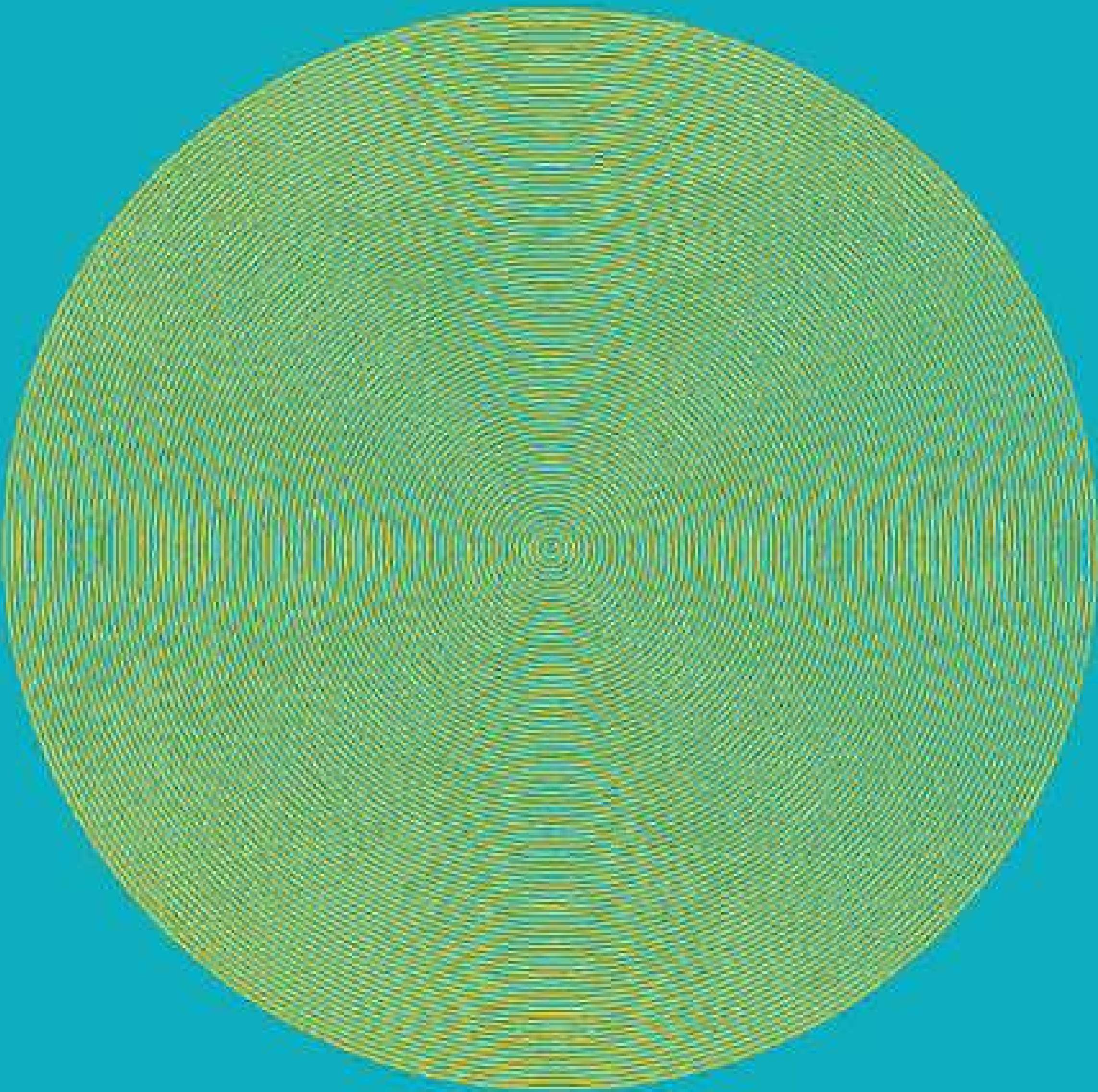
Kinetic Energy Storage

G Genta



Butterworths

Kinetic energy storage

Theory and practice of advanced flywheel systems

To Franca and Alessandro

'From this follows that if the (Supreme) Intelligence refrained from moving the heavens they would continue to move, that is, to rotate, for some time thanks to the very virtue that assigns and continues circular motion, as does in the case of the potter's wheel, which continues to spin for some time after its prime mover has ceased to move it.'

Franciscus de Marchia, 'In libros sententiarum', 1323

'And there is an experiment: if you spin fast a large and heavy grinding wheel of a blacksmith and then you refrain from moving it, it continues to move owing to the impetus it acquired; moreover you could not stop it in a short time, but that impetus would be slowly and continuously dissipated owing to the resistance due to the gravity of the wheel until the motion stops; and perhaps if the wheel would last forever without any reduction or alteration and if there were no resistance to corrupt that impetus, the wheel would be moved for ever by it.'

Johannes Buridani, 'Quaestiones super libris quattuor de caelo et mundo', c. 1340

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Preface

Flywheels are among the most ancient mechanical devices known to man, and the use of the wheel for storing energy seems to be older than its use as a means of supporting vehicles.

It was, however, only since the early 1970s that the awareness of the growing costs of energy and of an eventual shortage of fossil fuel supplies engendered an interest in energy conservation and, in consequence, in energy storage devices, among which are flywheels.

Research on 'advanced' flywheels which can store as much energy per unit mass as other types of energy storage device has been undertaken in many countries, often supported by public funds.

This book is an attempt to systematize the results of research and to present an up-to-date 'state of the art' review of the subject, which the author feels is justified for the following reasons.

Firstly, there is no agreement on the practicability and the future possibilities of flywheel energy storage from either the technical or the economic viewpoint. Over-optimistic statements found in scientific literature often vie with unjustifiably pessimistic views. There is therefore a need to present unbiased information to help the designer or manager to appraise these devices in an informed and systematic way, though the author is bound to admit that his own moderate optimism on the future of flywheel devices may at times colour these pages. The reader cannot fairly expect otherwise.

Secondly, flywheel technology is a complex subject and, as far as high energy density flywheels are concerned, a highly specialized one, and it is not unusual to hear false statements or incorrect interpretations of recent results from otherwise well-informed persons. Nor should it be forgotten that flywheels are, and have been for centuries, a fundamental element of many machines. Their design is often regarded only in their ability to supply the required inertia to a rotating part without much concern for other characteristics.

Some of the results of the work aimed to produce viable flywheel accumulators can also be used to refine the design of flywheels used in 'conventional' applications, and so upgrade their performances.

The aims of this book are consequently twofold:

(i) to assemble information from various sources for those who have to decide the pros and cons of the various types of kinetic energy accumulator for given applications, and for the designers who must implement that decision.
(ii) to reveal to designers of rotating machinery of all types the full potential of modern flywheel technology.

Flywheel technology involves many aspects which belong traditionally to other disciplines. Thus the author's own technical background might be excused for any emphasis on the design of the rotor and its dynamic behaviour, while other perhaps not less-important elements receive less attention, consistent also with the need for reasonable economic constraints on the length of this book. An extended bibliography is intended for those who wish to delve deeper into these aspects.

The author expresses his sincere thanks to all persons and institutions without whose co-operation these pages would not have been written, and particularly to his colleagues and technicians of the Dipartimento di Meccanica of Politecnico (Technical University) di Torino, who took part in the research work on flywheel technology and whose suggestions, criticism and general exchange of ideas contributed much to this work; also to the Italian National Research Council (C.N.R.) who sponsored the work and under whose founding the research is proceeding and whose contribution is much appreciated. Thanks are also due to those who have participated in the research, and their research managers, in particular Dr P. Motta of CISE (formerly with Industrie Pirelli S.p.A.) whose co-operation was essential for the experimental part of the work, and to other research institutions with which a constant exchange of opinions and results has proven invaluable, especially to Dr D. W. Rabenhorst, whose fruitful and objective work at the Applied Physics Laboratory, Johns Hopkins University, has proved to be an invaluable source of ideas and encouragement.

The author also wishes to register his appreciation of the help given by Dr T. M. Barlow of Lawrence Livermore National Laboratory, who is responsible for the Mechanical Energy Storage Technology Project, sponsored by the US Department of Energy, and who provided the author with all the printed matter issued on the subject.

Last but not least, a sincere thanks to my wife, Franca, both for her encouragement and for having done the tedious work of revising the manuscript.

G. Genta

Contents

Symbols

a	acceleration; effective molecular diameter
c	radial clearance between the flywheel and the housing; damping coefficient
d	distance
d_m	average diameter of a bearing
d_s	diameter of a shaft
e	energy; base of natural logarithms
f_0	rolling resistance coefficient (at zero speed)
g	acceleration of gravity
h	disc thickness at radius r
i	imaginary unit ($i = \sqrt{-1}$)
k	ratio between the specific heat at constant pressure and the specific heat at constant volume; stiffness
m	mass; mass of the molecules
m_a	'apparent' mass of the vehicle
m_m	molecular mass
p	pressure
q_i	generalized coordinate
r	radius
r_i	inner radius
r_I	radius of inertia
r_o	outer radius
t	time
u	radial displacement; eccentricity
v	mean velocity of the molecules
z	complex coordinate
B	parameter for constant stress discs and bars; ratio $\sigma_{\theta}/\sigma_{zr}$
C_m	aerodynamic drag torque coefficient
C_x	aerodynamic X-force coefficient
E	Young's modulus
G	shear modulus
H	angular momentum

I	area moment of inertia
J	mass moment of inertia
K	shape factor, term expressing the dependence of the rolling resistance from the speed; Boltzmann constant
K_n	Knudsen number
M	moment; Mach number
M_a	aerodynamic drag torque
N	number of spokes
N_A	Avogadro's number
P	power
Q_i	generalized force
Q_{ij}	element of the plane stress stiffness matrix
R	universal gas constant
R_c	inner radius of the containment ring
R_e	Reynolds number
S	area; pumping speed
S_{ij}	element of the elastic compliance matrix
T	(absolute) temperature; period
T_s	Sutherland constant
V	volume; velocity
V_f	fibre content of a composite material (in volume)
V_s	velocity of sound
α	thermal expansion coefficient
β	ratio r/r_o
β_{ij}	element of the plane strain compliance matrix
γ	shear strain; ratio $\sqrt{Q_{\theta\theta}/Q_{rr}}$; structural damping factor (loss factor)
δ	ratio r_i/r_o; radial clearance in squeeze film dampers
ε	normal strain; non-dimensional eccentricity
η	efficiency; viscosity
ϑ	polar angle
λ	mean free path of the molecules; whirl speed
μ	ratio E_θ/E_r
ν	Poisson's ratio; kinematic viscosity
ζ	velocity factor
ρ	density of the material
ρ_a	density of air
ρ_g	density of the gas in the flywheel container
σ	normal stress
σ_e	equivalent stress
σ_u	ultimate strength of the material
σ_y	yield stress
τ	shear stress; transmission ratio
ϕ	complex variable
χ	non-dimensional coordinate
ω	angular velocity (spin speed)
χ	dynamic unbalance
Ω	angular velocity
$(\;)_{,x}$	differentiation with respect to the variable x
$\times$	cross product of vectors

Subscripts

c	circumferential direction
e	equivalent
f	flywheel, fibre
i	inner
m	matrix
o	operating; outer
o, o	operating overall
r	radial direction; containment ring
s	shaft
u	ultimate
u,f.	ultimate flywheel
z	axial direction
L	longitudinal direction of unidirectional composite materials
T	transverse direction of unidirectional composite materials

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1 Historical background

1.1 Philosophers and flywheels

Ancient Mechanics[*] was dominated by the idea that a body could be maintained in motion only if a suitable force was applied to it. Aristotle thought that the velocity of a body was proportional to the force applied.

Some major difficulties arose from this idea, particularly when the motion of projectiles or the free falling of bodies had to be explained. The ideas on these subjects were quite confused and contradictory explanations were often propounded by the same author. As an example, it was necessary to assume the existence of a *prime mover* which sustained the motions of celestial and earthly bodies.

Aristotle, in his fourth book of *Physica*, explains the motion of projectiles by assuming that the air, rushing behind the projectile in order to fill the empty space left by its motion, exerts a continuous push.

The problems, however, remained unsolved and even in the 6th century AD the grammarian Johannes Philoponus in his comment to the *Physica* of Aristotle was able to offer an equally spurious counter-proposal that in being thrown, a certain undefined 'force' is transferred to the projectile, which was able to sustain its motion for some time.

For ancient philosophers, translatory and rotary motion were different phenomena, and often the explanations offered for one were not applied to the other.

The two theories attributing the causes of motion to the medium and in the object itself remained for centuries, sometimes battling with each other and sometimes with attempts to integrate them in a single theory.

Ibn Sina (Avicenna, 1037), in his comments on Aristotle, states that this 'inclination' (in Arabic, 'mail') transferred to the projectile in being thrown

[*] A detailed survey of ancient and medieval mechanics is beyond the scope of this section. Only some points that the author found of interest in the context of flywheel machines will be reported. A primary source of information is, for this first section, the book: Clagett M., *The Science of Mechanics in the Middle Ages*, University of Wisconsin Press (1959).

would last indefinitely were it not for the drag of the medium in which motion takes place.

The same opinion was put forward by Jean Buridan, the famous rector of the University of Paris in the 14th century, in his theory of impetus. He suggests that there is no need to think that God continues to move the celestial bodies as it is simpler to assume that He gave them the required impetus when they were created, there being no resistance to motion in the perfect celestial world.

He takes as an example of motion, which can be explained only with his theory, the flywheel, or the 'rotula sive lignea sive plumbea tornatilis', which maintains its velocity for a long time without changing its position in space. He also affirmed that the impetus is proportional to mass and speed and says that the difficulty found in slowing a heavy grinding stone is due to mass and speed of rotation.

Buridan seems to think that rotational and translational 'impetus' are essentially the same thing. Other philosophers held the opposite opinion (e.g. Henry of Assia, who worked in Paris between 1360 and 1380). About the fact that only external resistance slows down a body, others thought that the impetus could not last for ever even in the absence of forces which resist motion. Of this opinion was Oresme, another famous disciple of Buridan.

G. B. Benedetti (1585) states that a wheel cannot continue its rotation indefinitely even in the absence of friction as, normally, the impetus would make the parts which compose it move in a straight line (i.e. tangentially). The constraint which compels them to follow a circle has the effect of diminishing their impetus. He uses this tendency of the parts of the wheel to move in a straight line to explain the stability of a top or other rotating body.

It is interesting that, while ancient science failed to explain inertial motion, it was well aware of centrifugal stressing of rotating bodies and realized the danger of the bursting of rotating objects such as grinding wheels.

Copernicus opposes the opinion of Tolomeo that if the earth were to rotate it would burst, by observing that the sky moves faster, being of greater diameter and would more likely burst if the circular motion were imparted to it. A very interesting discussion on the stability of the sky can be found in Copernicus. He says that it is absurd to think that the sky does not collapse because it rotates as this equilibrium would be unstable and it would either collapse or grow infinitely. The fact that things are unable to remain whole if they rotate too rapidly is assumed from experimental evidence. Only when the law of inertia was proposed by Galileo and modern mechanics was started by Galileo and Newton could the way in which inertial devices work be explained in a satisfactory way.

1.2 From prehistory to the Roman civilization

The failure of ancient thought to recognize the basic principles of motion did not, however, hamper the development of devices which exploit the inertia of bodies. It must be remembered that in the ancient world, science, i.e. philosophy and technology, were far more separated than in modern times and, besides this, inertial devices were already well established thousands of years before attempts were made to explain rotary motion.

Inertia of translation was exploited far earlier and to a larger extent than rotary, as it is far easier to move an object on a more or less straight line than to spin it.

The principle of the hammer is so easily grasped that simple tools such as sticks are used even by lower animals. Some examples are reported in the well-known *History of Technology* edited by C. Singer, E. J. Holmyard and A. R. Hall.

It was, however, only with the evolution of man that the use of simple tools became common, but hundreds of thousands of years were to elapse before rotary inertia could play a role in human life. The first tool using rotary motion was probably the drill, in its two main applications, boring and lighting fires. But the hand drill, like its immediate successors, the strap drill and the bow drill, which are still widely used by some primitive peoples, did not need the addition of a flywheel to work properly.

The first object which made use of rotary inertia is the spindle whorl. Spinning, i.e. drawing fibres and twisting them to form threads, was started by rolling the fibres between the palms of the hands or the hand and another part of the body. Eventually the use of a stick to wind the thread was established and the spindle evolved from it. If the spindle was rotated directly by hands or rolled on the thigh there was no need to increase its inertia. A third method of spinning, however, evolved in which the spindle was suspended by the thread itself. A length of fibres would be drawn, twisted slightly and fastened to the spindle. The latter was then rotated by hand and dropped while the fibres were regularly paid out. The spindle maintains its rotation under its own inertia and therefore utilizes a flywheel effect.

If the spindle is simply a stick of wood, the flywheel, namely the whorl, can be a small piece of wood, stone, pottery, metal, glass, or bone, with a central hole. More modern forms of integral spindles appeared later.

A spinning whorl found in the excavation of Pan Po prehistoric site near Xian is shown in *Figure 1.1(a)*. It is difficult to assess when this device came into use but V. G. Childe suggests it happened in the sixth millenium BC or later.* Archeological evidence from the third millenium BC onwards shows that spindles with whorls were very widespread from Swiss lake-village culture to China, and in many other Neolithic and Bronze Age cultures. In Troy excavations more than eight thousand spindle whorls have been found. It is very noteworthy that no spindle whorls have been found in Peru, and that in that country natives still spin with a small unweighted stick.

Another application of the flywheel which appeared in very ancient times is the potter's wheel. It seems that the first potters' wheels appeared about two millennia later than the first spindles and this gap is easily explained remembering the major difficulties linked both with the greater size and energy stored and the need for some sort of bearing, as potters' wheels must rotate about a fixed axis. The first potters' wheels were built from wood, and consequently direct archeological evidence is scanty. Often their use can be inferred from stone bearings found on sites, but often these are not identified among the quantity of various other stones. The use of the potter's wheel has to be inferred from the style of the pottery. It is easy to say whether it was built up

* Childe, V. G., 'Rotary Motion', in *A History of Technology*, edited by C. Singer, E. J. Holmyard and A. R. Hall.

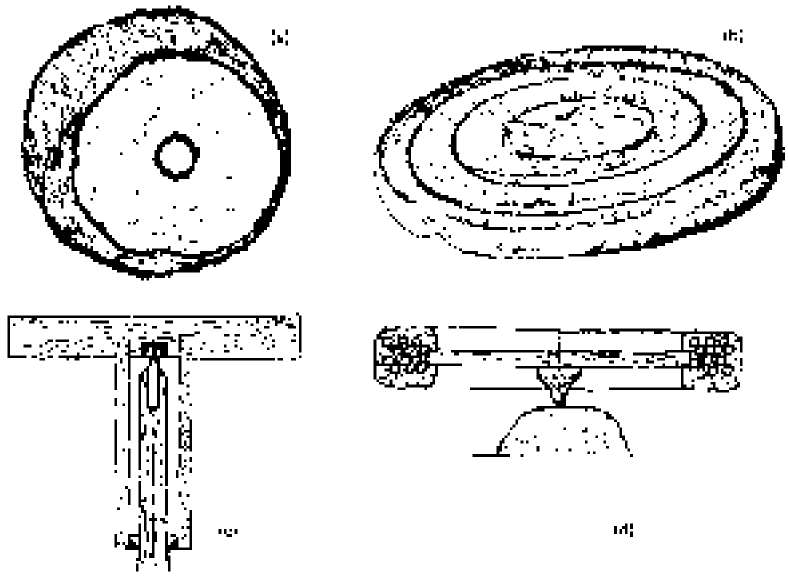

Figure 1.1 (a) Small spindle wheel (only a few centimetres in diameter) found in the excavation of Pan Po prehistoric site near Xian, central China. This tiny flywheel is thought to be 6000 years old.
(b) Potter's wheel found in South Mesopotamia. The wheel, from about 2000 BC, is made of clay and has a diameter of ~900 mm and a thickness of ~80 mm.
(c) and (d) Bearings for potter's wheels.
In (c) the wheel is supported over a fixed pivot and is stabilized by a radial bearing carried by a rotating hollow cylinder. The wheel is from Japan and the bearing in the wheel is a porcelain cup.
In (d) the wheel carries a pivot, which rests on a piece of hardwood or a stone embedded in the clay base. The wheel is from India and is made of bamboos covered with bonded clay. The spokes are of wood.
(a) is sketched from a picture taken at Pan Po Museum while (b), (c) and (d) are from Singer et al., 'A History of Technology'.

by hand, perhaps on a turntable, or actually was thrown on a fast-spinning wheel. Perhaps for this reason, the archeological evidence indicates that the potter's wheel preceded the wheeled vehicle. The first findings are from Sumer and Susiana, and date from between 3500 and 3000 BC.

The potter's wheel must not be confused with the turntable, although it might be a development of the latter. When making a pot on a turntable the potter will add clay, often in a form of a small cylinder, while slowly rotating the table by hand or foot. With the potter's wheel, on the other hand, a lump of clay is placed at the centre of the wheel and is set spinning by a strong jerk on a stick which engages in a hole in the rim or on a strap wound around the wheel or its axle. The wheel could also be spun by the hand or foot of the potter or of one of his assistants. The lump of clay could thus be put at the centre of the wheel and shaped directly by hand while centrifugal forces lead the material to its final form.

The wheel needs to maintain its rotation for long enough to complete the operation of shaping the pot, whether in one or more stages. It is therefore a true flywheel, and the energy stored in it is by no means negligible. V. G. Childe suggests that the wheel should maintain its rotation for five to seven minutes with an initial angular velocity of 100 rev/min. The energy stored at this velocity by the wheel found at Erech, Mesopotamia, in the burial place of a potter and dating back to about 2000 BC (*Figure 1.1*), would have been about 500 J (0.14 Wh).

The wheel, made of a disc of wood, stone or clay, can rotate on a fixed pivot which fits in a socket in its underside as shown in *Figure 1.1(c)* where a 'radial bearing' is added to increase the low-speed stability. Alternatively, a short pointed stub shaft can be fixed to the lower surface of the disc (*Figure 1.1(d)*). The last solution is particularly unstable at low speed, as it relies on the gyroscopic effect of the flywheel to keep its plane of rotation in a horizontal position.

The wheel of *Figure 1.1(d)* is particularly interesting, as the use of a rim made from a unidirectional material (bamboo) wound in hoop direction embedded in a matrix (clay) clearly foreshadows modern composite flywheels.

An improvement of the simple wheel is the so-called 'foot wheel', as the ones described in far later times by Piccolpasso (*Figure 1.2*) or Agricola. It is difficult to assess when this type of wheel became used, but it is thought that it was in use in the Middle East before 2000 BC. The flywheel was set in a shallow pit with the axle pivoted at the bottom and radially supported at ground level by a plank with a round hole. The potter was seated on a stone at the edge of the pit. This solution is the best also from the safety viewpoint, as a rotor burst could easily occur with clay discs.

The construction of suitable bearings would have been quite a problem and

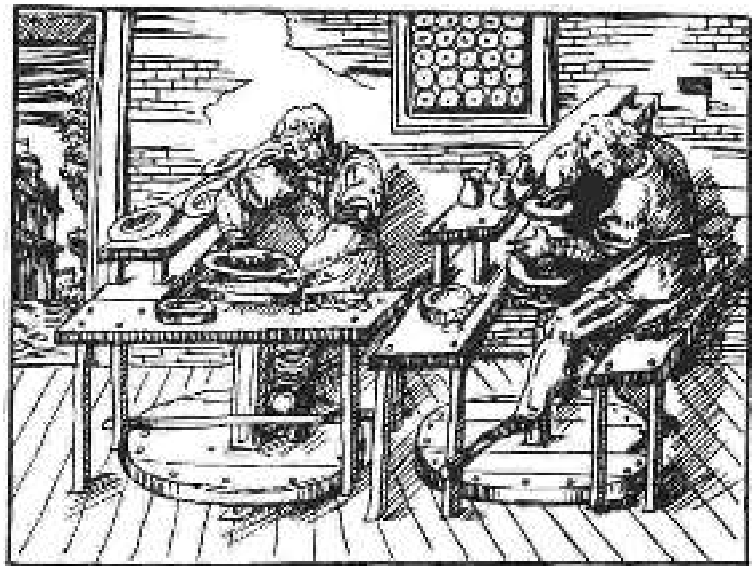

Figure 1.2 Two potters using foot-wheels. From an Italian treatise on pottery written by Cipriano Piccolpasso about 1550 AD. The constructional details of the wheel are interesting: it is built from a sort of plywood and can be considered as a forerunner of modern plywood or laminated composite disc flywheels

was sometimes solved using stone sockets and pivots even if simple hardwood bearings were more common. In China a porcelain cup is still common for the socket. Fats, vegetable oils or bitumen might have been used as lubricants, and there is some evidence of the use of the latter.

The potter's wheel played a certain role in the development of civilization. With it, a professional potter could shape a vessel in a very small fraction of the time required by a housewife to perform the same job. Its diffusion required certain economical and social conditions: only a professional craftsman was able to build and to operate such a machine. A social surplus had to be available in order to support the potter and his family and, as pots could not be easily transported over long distances, this surplus had to be produced locally. As it can be assumed that these conditions could be met only in villages of at least two hundred households, it is clear that only in the agricultural civilization of Egypt, India and Mesopotamia the potter's wheel could be used for centuries.

Archeological evidence for the spread of thrown pottery and, consequently, the wheel itself, indicates its beginning in Sumer at between 3500 and 3000 BC, spreading to the Mediterranean coast: Syria, Palestine (3000 BC), Egypt (2750 BC), Harappa and Mohenjo-Daro (2500 BC), Crete (2000 BC), Greece (1800 BC), Southern Italy (750 BC), Central Europe (400 BC), England (50 BC). In the Americas it was unknown along with the wheeled vehicle, until European colonization.

The use of some other machines incorporating a flywheel of some sort, particularly for boring and grinding, can be deduced from remains and from paintings. But often the archeological evidence is scarce and the drawings far from clear.

Flywheels were used in Egypt in the 15th century BC for twisting strips of leather to make ropes for shipbuilding, according to a painting found in the Dynasty XVIII tomb of Rekhmire (*figure 1.2*). A massive flywheel (perhaps of stone) with a handle was connected to a sort of spindle through which the seated man feeds the leather strips to be twisted. Another man keeps the rope in tension while a third turns the handle.

The idea of this device might have come from the spinning of thread, and the

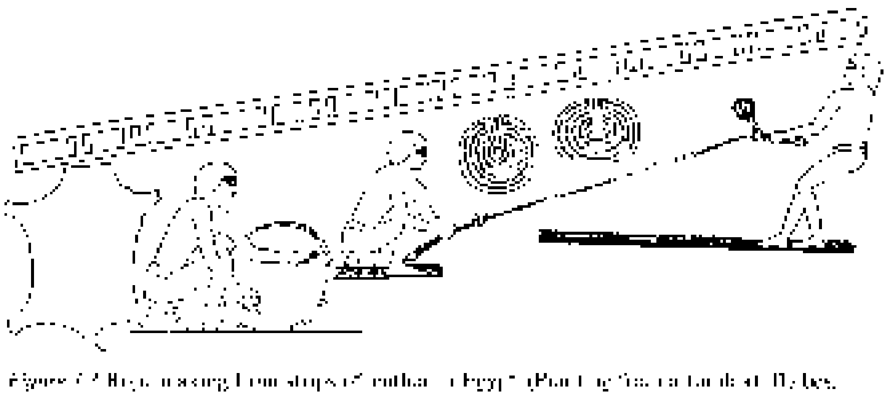

Figure 1.2 Rope making from strips of leather in Egypt (Painting from a tomb at Thebes, c.1450 BC). A flywheel is used to assist the rotation of the spindle (Singer et al., A History of Technology).

use of a flywheel of this size might derive from the potter's wheel. Sometimes a 'flywheel' can be simply formed by a stick with two stones at its ends which is rotated about its midpoint.

1.3 From the beginning of the Christian Era to the Industrial Revolution

A new application of the flywheel appeared in Roman times: the pump-drill (*Figure 1.4*). This device works by pushing the crossbar downwards, causing the string to untwist. A flywheel is necessary both for steadying the motion and

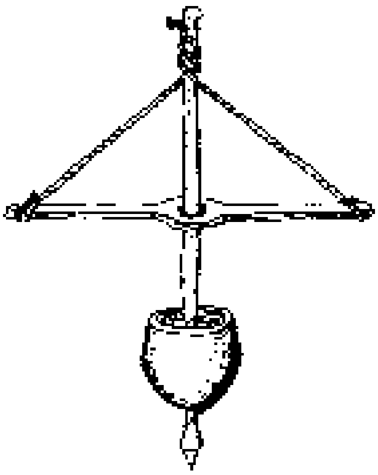

Figure 1.4 Pump-drill from Nias, Indonesia. The flywheel is a coconut filled with stones. (Singer et al. 'A History of Technology')

for twisting the string again in order to have it ready for the next stroke without having to twist it by hand. This is particularly important when the drill is used to make fire, and a fast and continuous operation is required. This drill seems to be an important improvement over the strap drill and the bow drill as all the forces exerted on the bit shaft are parallel to its axis and no transversal force is present. A radial bearing steadies the shaft and only one operator is needed to perform the work. If he is skilled enough, a fast rotation can be achieved and the pressure of the bit can be quite uniform during the downstroke. The rotation is intermittent, with frequent reversal of direction – this is common to all ancient drills and lathes.

In the first centuries AD, mechanical prime movers appeared. The water mill was introduced in the 1st century AD and was in widespread use by the 4th century. The windmill appeared in Western Europe about a thousand years later and is widely believed to have been used in eastern countries even earlier. Water mills were first used for grinding corn, in which application it was unnecessary to increase the inertia of the rotating parts. The inertia of the water wheel and of the grinders was sufficient. Biringuccio, in his *Pirotechnia* (1540), describes a method for drawing iron wire using the power from a water wheel (*Figure 1.5*). Even in such applications where inertia of rotation is

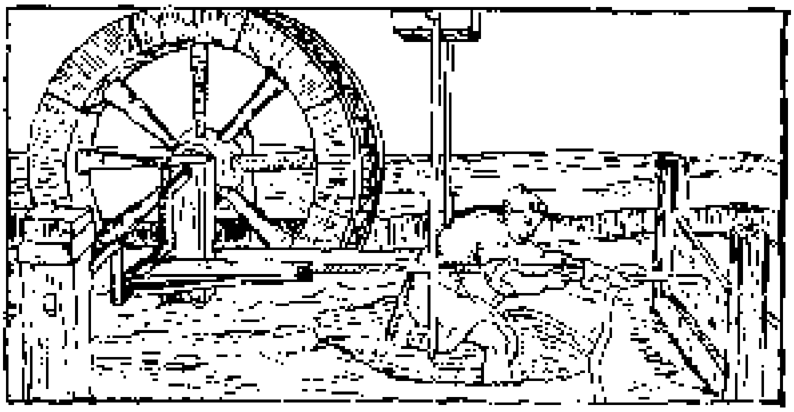

Figure 1.5 Drawing heavy iron wire. The workman is seated on a swing which moves with the crankshaft is operated by the undershot wheel. He moves the tongs forwards on the forward motion of the swing when the rope is slack. A stirrup causes them to grip the wire when the rope is in tension. Note the absence of a flywheel: the high value of the moment of inertia of the wheel makes it unnecessary. (From Biringuccio, Pirotechnia, 1540)

essential, the wheel inertia and perhaps the high torque output at low speed of the wheel, obviated the need for additional inertia.

A water wheel of 3 m diameter could have a mass of about 1500 kg and a corresponding moment of inertia of the order of 1000–1200 kg m². At a speed of 50 rev/min the energy it stored was in the range of 8–10 Wh.

Similar examples are to be found among windmills. In the so-called 'giant wheel of Pan Po', a sail wheel used in China with the aim of raising water from a ditch, heavy stones were added to the rim in order to increase the inertia of the wheel (*Figure 1.6*).

A detailed description of this machine is given by N. V. Gulia [76–13], which, though lacking chronological details, explains how inertial masses might have been used to steady the rotation in gusty conditions. Apart from the use of the inertial masses, the dimensions alone make this an interesting piece of machinery. It is reported to have been 'as high as four men above the ground and two men below'. The author is, however, not sure whether the inertial masses were used to even out the rotation in gusty conditions or more simply for very short-term perturbations like the ones caused by uneven friction or by the contact of the buckets with water. Various types of sail wheels with vertical or horizontal axis are used even today in China, but no mention is made of inertial masses.

In the Middle Ages the power supplied by men and animals to rotating machinery was often applied through 'squirrel cage' wheels or wheels moved by the hind legs of the animal while the forelegs rested on a fixed surface, as shown in *Figure 1.7*. The drawing from a manuscript of Francesco di Giorgio Martini (1439–1502) describes a corn mill.

The big wheel had significant inertia, and what has been said about water and wind mills also holds for these devices.

There is, however, an important exception, linked with the use of gearwheels, which appear with increasing frequency in this period. Owing to the poor kinematic characteristics of the gear wheels of various types then in

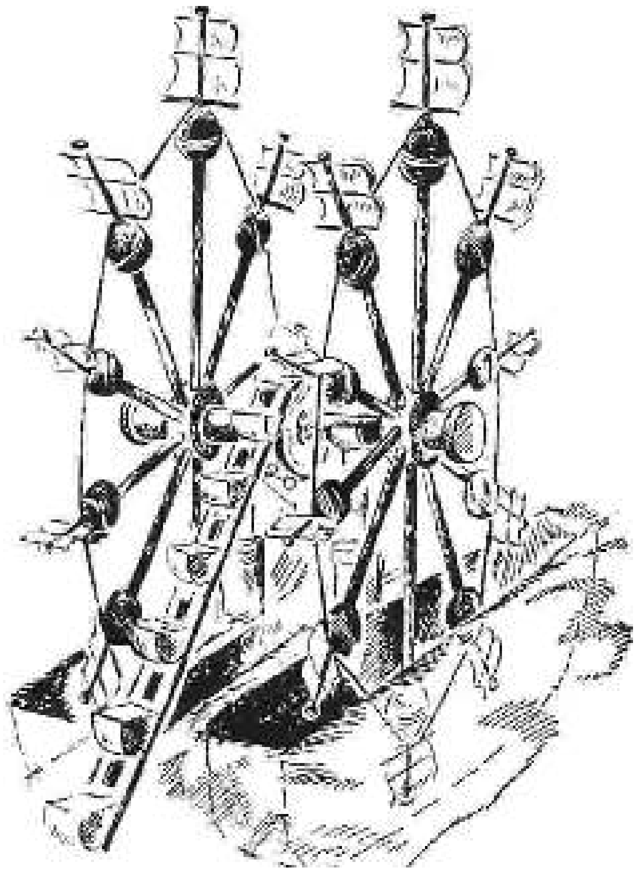

Figure 1.7 A Chinese sail wheel incorporating inertial masses of stone. This is 'the giant wheel of Pan Po'. The supports have been omitted. (Gille [76.13])

use, high moments of inertia were often added to all the shafts, except the ones which carried the grinders and the water wheel. This was usually done by building heavy gear wheels (which may have been done unconsciously) but in some machines flywheels were purposely added, at least on some shafts.

The introduction of the crank to convert rotary into linear motion and vice versa is usually ascribed to Heron (1st century BC?). If it is the connecting rod which gives motion to the crank, the need for an adequate moment of inertia becomes of paramount importance. This was clearly understood, as testified by the discussions of the dead points in crank mechanisms.

As an example of a treadle operated mechanism, a lathe from a manuscript of Leonardo da Vinci is shown in *Figure 1.8*.

Hand mills which operated through a crank were widely used in emergency conditions, such as sieges. As an example, a drawing from a manuscript of Francesco di Giorgio has been reproduced in *Figure 1.9*. Some interesting constructional details of this flywheel will be discussed later.

The spinning-wheel, which also appeared in the Middle Ages, is another example of a machine operated by a crank through a pulley with a high inertia

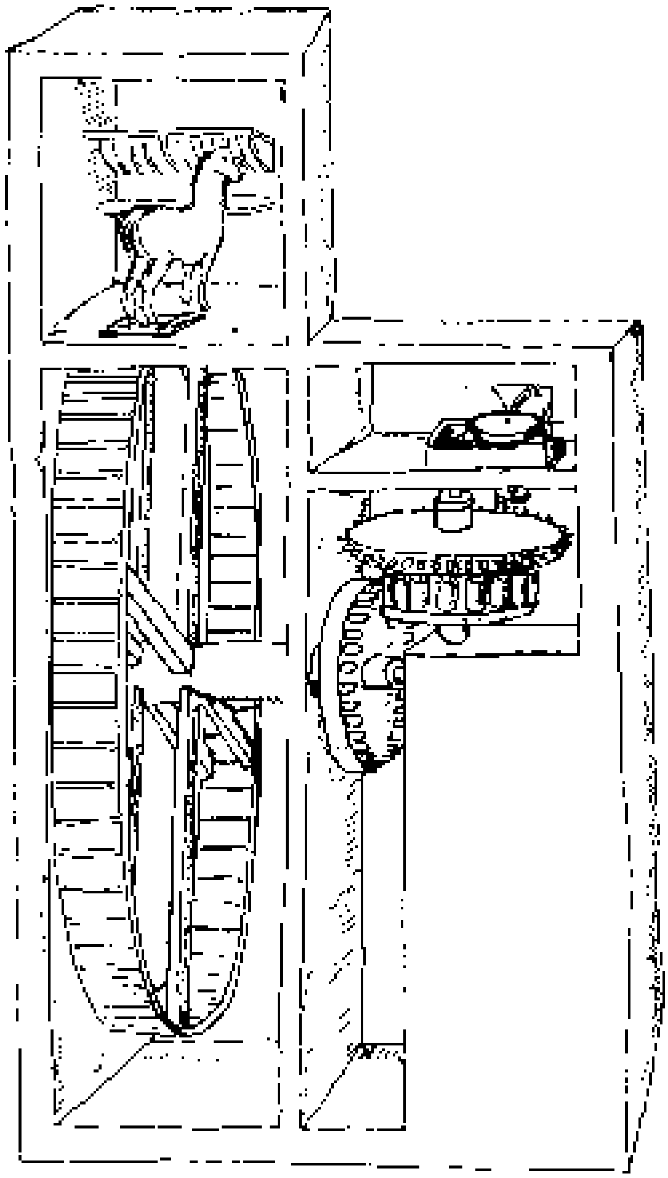

Figure 1.9 Mill operated by a horse through the cog-wheel and lantern (from the Trattato di Architettura by Francesco di Giorgio Martini)

which can be considered a flywheel. An illustration from the *Luttrell Psalter* given in *Figure 1.10* shows the simplest form of spinning-wheel as used in the 14th century. Later improvements were particularly devoted to obtaining a continuous operation of the machine by adding a flyer, which makes simultaneous spinning and winding possible. The drive system did not change and spinning wheels from the 19th or even 20th century do not differ in this respect from those of the Middle Ages.

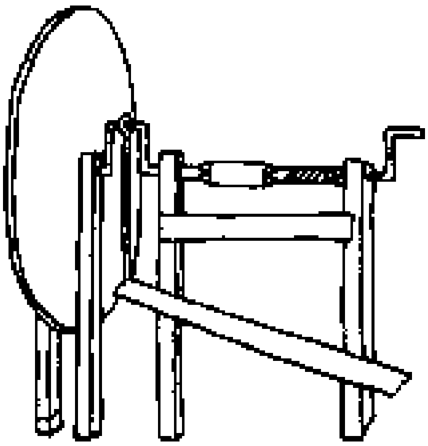

Figure 1.8 Lathe driven by a treadle through a crank mechanism. (From a manuscript of Leonardo da Vinci.)

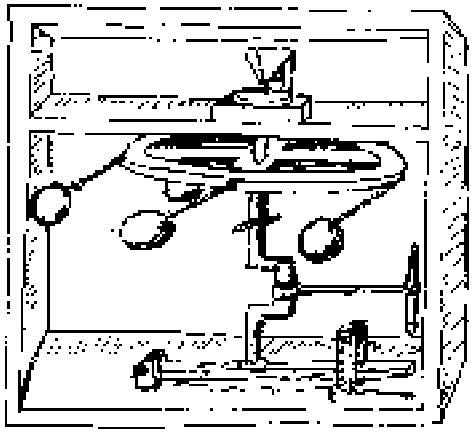

Figure 1.9 Hand mill from a manuscript of Francesco di Giorgio Martini (see *Figure 1.7*). Note the wrong position of the ball on the right

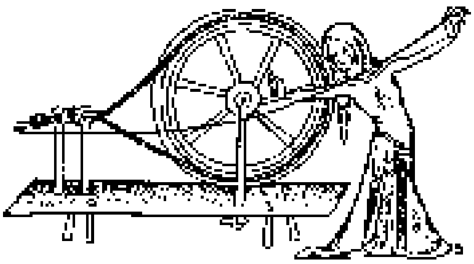

Figure 1.10 An illustration from the Luttrell Psalter (c. 1338) showing a spindle wheel

Flywheels were used in lifting devices, as shown in many drawings. Agricola, in his *De re metallica* (1556), describes a bucket water hoist which incorporates a flywheel. Lifting devices incorporating flywheels were probably common at earlier dates.

In the centuries before the Industrial Revolution, there was a gradual increase in the spin velocity of rotary machines. Apart from spinning, in which