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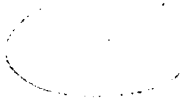
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# INTRODUCTION TO SUPERCONDUCTIVITY

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Second Edition



**Michael Tinkham**

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and*

*Gordon McKay Professor of Applied Physics  
Harvard University*

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## **INTRODUCTION TO SUPERCONDUCTIVITY**

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A native of Wisconsin, **Michael Tinkham** received an A. B. degree from Ripon College and his M. S. and Ph.D. from MIT. After a postdoctoral year at the Clarendon Laboratory in Oxford, he spent 11 years teaching at the University of California, Berkeley, before moving to Harvard in 1966, where he is now the Rumford Professor of Physics and Gordon McKay Professor of Applied Physics. Over the years, he has spent sabbatical leaves at MIT and at the University of Paris, Orsay, as a Guggenheim Fellow; at the Cavendish Laboratory in Cambridge University as an NSF Senior Postdoctoral Fellow; at the Institute for Theory of Condensed Matter in Karlsruhe, Germany, as a Humboldt Prize Fellow; at the University of California in Berkeley as a Visiting Miller Professor; and as a Visiting Professor at the Technical University of Delft, the Netherlands.

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Author of over 200 research publications, he has written three previous books: *Group Theory and Quantum Mechanics*, *Superconductivity*, and the first edition of *Introduction to Superconductivity*, which has been translated into Russian, Japanese, and Chinese.

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# PREFACE

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The first edition was written after the surge of activity in the 1950s and 1960s, in which the Bardeen-Cooper-Schrieffer (BCS) and Ginzburg-Landau (GL) theories were assimilated and used to develop what was referred to in the first paragraph of that edition as a “remarkably complete and satisfactory picture” of superconducting phenomena. It was written with the aim of presenting that picture to its readers, and it has performed that function well during the subsequent 20 years. Very little in the first edition has proved to be incorrect or misleading. Why, then, make the very substantial effort to prepare a second edition?

The major impetus has, of course, come from the discovery of high-temperature superconductivity, and the immense body of activity which this has spawned. This phenomenon is not only of great interest for its own sake, but it has also provided us with a broader perspective for understanding the properties of the classic superconductors as well. Another impetus was my desire to include an introductory survey of the many developments in classic superconductivity over the past two decades. Inevitably, I have given some extra emphasis to areas in which my own group has been active, such as Josephson junctions, single-electron tunneling effects, and nonequilibrium superconductivity, but these are also topics of broad interest. Finally, I wanted to take advantage of the 20 years of teaching experience with the first edition and to try to produce a new edition which is somewhat more “user friendly” for the beginner in the field.

Rather than reviewing the whole book, let me simply highlight the changes from the first edition. First, before attacking the intimidating BCS theory, I have introduced a new Chap. 2, which, drawing on the latter part of the old Chap. 3, treats example applications of the elementary London theory of the electrodynamics of superconductors, including diamagnetic screening, the intermediate state, and high-frequency absorption. For many practical purposes, this is the most important level of understanding, and it seems counterproductive to delay discussing these examples until after BCS, as was done in the first edition.

Second, although the old BCS chapter has been renumbered as Chap. 3, it is largely unchanged except for the addition of a new Sec. 3.11 on the penetration

depth. This new section collects the essential material from the first part of the old Chap. 3 on the implications of the BCS nonlocal electrodynamics for the determination of the effective *local* penetration depth used throughout the rest of the book. At the same time, the less essential mathematical details of this discussion are either relegated to the appendix or omitted entirely, because the *conceptual* importance of nonlocality is now well established; but nonlocality is not relevant to the high-temperature superconductors or the dirty materials of technical importance for magnet wire. Instead, there is some new discussion of how  $\lambda$  is measured experimentally and other new material introducing the perpendicular penetration depth  $\lambda_{\perp}$  of thin films.

The first really major change has been to replace the old Chap. 6 on the Josephson effect by two entirely new chapters. In this expanded and modernized treatment, Chap. 6 covers the more basic principles and applications, making use of the popular “tilted-washboard” picture for discussing the RCSJ model, including  $I$ - $V$  characteristics of both under- and overdamped junctions. It also discusses Shapiro steps, photon-assisted tunneling, quantum interference, and sine-Gordon solitons in long junctions. This is followed by a modernized discussion of applications of Josephson junctions to SQUIDS, an entirely new discussion of *arrays* of Josephson junctions, and a brief discussion of superconducting tunnel junctions as high-frequency detectors.

The new Chap. 7 deals with the many special properties of the small, low-capacitance junctions made possible by modern nanofabrication techniques. In such junctions, the charging energy of single electrons is important and may dominate the Josephson coupling energy, so that the particle number becomes a better quantum number than the phase  $\varphi$ . This chapter deals with such topics as the Coulomb blockade, the single-electron tunneling transistor, and macroscopic quantum tunneling, as well as the importance of the damping of high-resistance junctions by low-impedance leads. Although these topics are currently the focus of intense research activity and can considerably deepen one’s understanding of the phase-number uncertainty relation in superconductors, this chapter can be skimmed over in a first course on superconductivity.

The second major change is the introduction of an entirely new Chap. 9, dealing primarily with the high-temperature superconductors. Because the fundamental mechanism responsible for the high  $T_c$  remains to be identified with certainty, we sidestep this question and emphasize instead the many properties of these materials which can be understood in the framework of the classic Lawrence-Doniach model of layered superconductors. These include the magnetic anisotropy and the implications of the flux-line-lattice melting transition for the resistive transition. Although its applicability is not restricted to the high-temperature superconductors, we next review the Larkin-Ovchinnikov model of collective pinning, including a short discussion of flux creep in this model and also in the vortex-glass and Boson glass models. The chapter concludes with a discussion of anomalous properties of the high-temperature superconductors which cannot be understood in terms of standard  $s$ -wave BCS superconductivity but, instead, suggest  $d$ -wave pairing.

The third major change is the addition of the new Chap. 11, dealing with nonequilibrium superconductivity, using a simplified version of the Schmid-Schön formalism to discuss the many implications of quasi-particle disequilibrium in driven superconductors. This chapter includes discussions of the *enhancement* of superconductivity by microwave radiation, by quasi-particle tunneling, and by dynamic nonequilibrium effects associated with a time-dependent energy gap. Also discussed here are phase-slip centers and the interconversion of normal and supercurrent by Andreev reflection at *NS* interfaces. The latter is a relatively old subject which, applied to more general configurations, has recently enjoyed a resurgence of interest.

The other five chapters have been left largely unchanged, not because they could not be improved but, rather, because doing so would have unduly, perhaps indefinitely, delayed completion of this new edition. Nonetheless, some changes were made. A number of new references were added to reflect the progress made in the intervening years. Also, brief discussions of a number of new topics were inserted, such as the Kosterlitz-Thouless resistive transition in two-dimensional superconductors.

While the expansion of the text which was required to include new developments will make it more useful as a reference for researchers, it also makes the book too long to be covered completely in a one-semester course. On the other hand, the instructors in such a course can take advantage of this plethora of material to pick topics to their own taste, leaving other topics to be pursued in individual study, perhaps leading to the preparation of a term paper.

McGraw-Hill and I would like to thank the following reviewers for their many helpful comments and suggestions in the early stages of the development of the second edition: Alex de Lozanne, Philip Duxbury, Richard S. Newrock, John Ruvalds, Mark Rzchowski, and Dale Van Harlingen.

Finally, I am pleased to acknowledge the assistance of many other colleagues in encouraging my efforts, in helping to guide the focus of the revisions, and in providing generous assistance in improving the quality of the presentation. I am uniquely indebted to Rick Newrock for his careful and speedy reading of chapter after chapter and for his extensive detailed criticism of the manuscript as it neared final form, which spurred me on to make many improvements and clarifications. It is, of course, impossible to acknowledge all those who have helped in many ways over the many years (and I apologize in advance to all those inadvertently omitted), but I should at least mention (in alphabetical order) Ryogo Aoki, Mac Beasley, Chuck Black, Greg Blonder, John Clarke, Dick Ferrell, Michael Flatté, Pierre-Gilles de Gennes, Rolf Glover, Ashraf Hanna, Jack Hergenrother, Marco Iansiti, Mark Itzler, Charlie Johnson, Teun Klapwijk, Kostya Likharev, Chris Lobb, Hans Mooij, David Nelson, Miguel Octavio, Dan Prober, Dan Ralph, Mark Rzchowski, Albert Schmid, Gerd Schön, Bill Skocpol, Mark Tuominen, and Valerii Vinokur. Of course, I cheerfully accept full responsibility for any errors or misunderstandings which may appear despite their assistance!

*Michael Tinkham*

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# SUGGESTIONS FOR USING THIS BOOK

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The first edition of this book was sufficiently slender so that most of its contents could be covered in a briskly paced one-semester course. This is no longer the case after the expansion required to bring the second edition up to date. The best choice of topics to cover will depend on the length of the course, the level of students, and the interest of the instructor. The following suggestions are offered as guidance in structuring a one-semester introductory course at the beginning graduate or advanced undergraduate level based on the material in this book.

The historical overview in Chap. 1 should be read primarily to provide a bird's-eye view of the subject for orientation. The discussion of electrodynamics in Sec. 1.3 can be used in an introductory lecture, with the rest of the material essentially deferred until treated carefully in later chapters. Chapter 2 presents a systematic treatment of the electrodynamics of classic superconductors, the property that gives the subject much of its interest and importance, at the simple, but very useful, level of the London equations. It should be covered carefully, except that Sec. 2.4 can be omitted to save time.

Although the discussion of the BCS theory in Chap. 3 is kept as simple as possible without loss of rigor, it is still the most technically intimidating chapter. Because of its importance, it should all be examined at whatever depth seems appropriate to the class. In an elementary course, one might focus on the key *results*: the ground state, the energy gap, the density of states, electron tunneling, and the penetration depth, skimming lightly over the rest.

Chapters 4 and 5 on the Ginzburg-Landau theory and type II superconductors are central and should be covered carefully, except for Secs. 5.7 and 5.8, which can be skimmed. Likewise, the Josephson effect, treated in Chap. 6, is fundamentally very important, but the more specialized Secs. 6.6 and 6.7 can be skimmed or omitted in a first course. The special features of very small Josephson junctions

treated in Chap. 7 are of considerable research interest at the present time, but this chapter can be omitted or skimmed in a first course, unless it is of special interest to the instructor. Similarly, much of the discussion of fluctuation effects in Chap. 8 is of somewhat specialized interest, but Secs. 8.1 and 8.2 on fluctuation-induced electrical resistance should be covered, and Secs. 8.3 through 8.5 should at least be skimmed. With the intense current interest in high-temperature superconductivity, much of Chap. 9 should be covered, at least at the qualitative level. The material from Secs. 9.6.3 through 9.8 is of more specialized interest and might be skimmed or omitted.

Finally, the material in Chaps. 10 and 11 considerably enriches our understanding of superconductivity by considering such topics as dirty superconductors, gapless superconductors, and time-dependent and nonequilibrium regimes of superconductivity. Unfortunately, time limitations in a short course will probably only allow these topics to be skimmed over lightly for the general ideas, with special attention to Sec. 10.1.

In summary, the more advanced and specialized material, which can be skimmed or omitted in a one-semester course, is that in Chaps. 7, 10 and 11, and in some of the latter parts in Chaps. 3, 5, 6, 8, and 9.

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# PREFACE TO THE FIRST EDITION

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This book has evolved from a set of lecture notes originally written for a graduate course at Harvard University during the fall term of 1969. They were subsequently rewritten during a sabbatical leave at the Cavendish laboratory in 1971–1972 and during a repeat of the course in 1973.

The objective of the lectures, and of this book, is to provide an up-to-date introduction to the intriguing subject of superconductivity and some of its potential applications. The emphasis is on the rich array of phenomena and how they may be understood in the simplest possible way. Consequently, the use of thermal Green functions has been completely avoided, despite their fashionability and undeniable power in the hands of skilled theorists. Rather, the power of phenomenological theory in giving insight is emphasized, and microscopic theory is often narrowly directed to the task of computing the coefficients in phenomenological equations. It is hoped that this emphasis will make the treatment more palatable to the experimentalist, and also complement the more generous coverage of the formal theoretical aspects of the subject in most books presently available. Finally, the author was motivated by the hope that if the theoretical techniques were kept as elementary as possible, the work might have more value to undergraduates and technologists with incomplete backgrounds in theoretical physics.

In a sense this book forms an updated and greatly expanded version of the Les Houches lectures of the author, written in 1961. However, so much development of the subject has occurred in the intervening years that these notes were really rewritten (twice) from start to finish. In the process, the author has drawn frequently on the excellent book of de Gennes, *Superconductivity in Metals and Alloys*, and on the two-volume treatise *Superconductivity* edited by Parks. There is little in the book which has not been published previously in some form, but



some topics—particularly fluctuation effects—have developed too recently to have appeared in previous books.

No attempt has been made to give an exhaustive or definitive treatment. Such a treatment required the two-volume Parks treatise mentioned above. Rather, the author has chosen to introduce the reader to a selection of topics which reflect his own focus on the electrodynamic properties of superconductors, which, after all, give the subject its unique interest. The time limitation of a semester lecture course provided unrelenting discipline in limiting the number of topics and the depth of treatment.

The book starts with an introductory survey which lays out the ground to be covered in the book, and gives some of the milestones in the historical development of the subject. The reader is advised to treat this as an overview only, intended to introduce concepts and language, with the detailed explanations to be developed in subsequent chapters. He definitely should not puzzle over issues which are only sketchily introduced at this point.

The second chapter is devoted to “basic BCS,” the microscopic theory developed by Bardeen, Cooper, and Schrieffer to explain the superconducting state. This theory is placed at the beginning because no serious discussion of superconductivity is possible without concepts derived from the theory. Unfortunately, this chapter has by far the most forbidding formal nature of any part of the book, but this should not be allowed to discourage the reader. Little use of the mathematical details will be made in the following chapters, and so this chapter can be skimmed for the general ideas (which are summarized in the concluding section), and referred to later if more detailed understanding of some particular point is required.

With Chap. 3, we move into the phenomenological level of treatment, which characterizes the rest of the book. First, the implications of the nonlocal electrodynamics in determining the effective penetration depth of a magnetic field into bulk and thin film superconductors are explored, the thorough discussion of the latter topic reflecting a historical interest of the author. A simplified discussion is then given of the intermediate state, in which superconducting and normal material coexist in the presence of a magnetic field.

Chapter 4 develops the Ginzburg-Landau theory from the same phenomenological point of view used by the original authors. The theory is then applied to an extensive catalog of classic problems: domain-wall energy, critical-current density, fluxoid quantization, critical fields of films and foils, the upper critical field  $H_{c2}$ , the Abrikosov vortex state, and the surface nucleation field  $H_{c3}$ . The concepts treated here underlie the subjects treated in the following chapters, in addition to illustrating the power of the Ginzburg-Landau approach.

In Chap. 5, the magnetic properties of type II superconductors are developed in some detail. After the equilibrium flux density has been worked out, attention is focused on the creep and flow of the flux under the influence of transport currents. In this way, insight is obtained into the considerations which limit potential applications of type II superconductors in high-field magnets. The chapter concludes with a discussion of the factors governing the design of superconducting

magnets to cope with time-varying fields, including the use of twisted multicore composite conductors to minimize ac losses while maintaining thermal stability.

Chapter 6 is devoted to the Josephson effect and macroscopic quantum phenomena. These subjects represent some of the purest and most fundamental aspects of superconductivity, yet also provide the basis for sensitive instruments which have revolutionized electromagnetic measurements. Both aspects are reflected in the treatment given; in particular, the detailed discussion of practical SQUID magnetometers is the first to appear in a textbook.

Although for years it was thought that the effects of thermodynamic fluctuations were unobservably small in superconductors, the advent of the superconducting detectors just mentioned has made it possible to observe such effects both above and below  $T_c$ . Chapter 7 surveys these phenomena in both electrical conductivity and diamagnetism. For example, it is shown how fluctuation effects put a limit (though an astronomical one) on the lifetime of "persistent" currents below  $T_c$ , and how they also give rise to "precursors" of superconductivity above  $T_c$ . Because this subject has flowered since the date of the Parks treatise, this book is the first containing a thorough discussion of this interesting and informative new aspect of superconductivity.

The final chapter is devoted to introductory discussions of three topics: the Bogoliubov method, gapless superconductivity, and time-dependent Ginzburg-Landau theory. These topics go beyond the elementary Ginzburg-Landau phenomenology and bring in more microscopic considerations. Yet the basic concepts and conclusions have been drawn inevitably into the discussions of the topics treated earlier; moreover, taken together, they lay the groundwork for work going on at the present frontiers of research. Hence, it seems fitting to close the book with a peek at these topics, where the last word is by no means in.

Finally, the author is pleased to thank the reviewers of the manuscript for constructive suggestions; the detailed reading of the final manuscript by Dr. Richard Harris is especially appreciated. The comments of students who have used the notes also were particularly helpful. The speedy and accurate typing of Miss Patricia McCarthy in preparing the final manuscript was an invaluable incentive to continued progress. More generally, the author wants to thank his numerous students, colleagues, and collaborators, especially in Berkeley, Orsay, Harvard, and the other Cambridge, for making his exploration of superconductivity the pleasure it has been. Although it would be impossible to list them all here, I cannot close this Preface without explicitly acknowledging numerous seminal discussions over the years with M. R. Beasley, J. Clarke, P. G. de Gennes, R. A. Ferrell, and R. E. Glover III. If this book serves to initiate others into the fascination I have found in this subject, it will have well served its intended purpose.

*Michael Tinkham*

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# **INTRODUCTION TO SUPERCONDUCTIVITY**

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# CHAPTER 1

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## HISTORICAL OVERVIEW

Superconductivity was discovered in 1911 by H. Kamerlingh Onnes<sup>1</sup> in Leiden, just 3 years after he had first liquefied helium, which gave him the refrigeration technique required to reach temperatures of a few degrees Kelvin. For decades, a fundamental understanding of this phenomenon eluded the many scientists who were working in the field. Then, in the 1950s and 1960s, a remarkably complete and satisfactory theoretical picture of the classic superconductors emerged. This situation was overturned and the subject was revitalized in 1986, when a new class of high-temperature superconductors was discovered by Bednorz and Müller.<sup>2</sup> These new superconductors seem to obey the same general phenomenology as the classic superconductors, but the basic microscopic mechanism remains an open and contentious question at the time of this writing.

The purpose of this book is to introduce the reader to the field of superconductivity, which remains fascinating after more than 80 years of investigation. To retard early obsolescence, we shall emphasize the aspects which seem to be reasonably securely understood at the present time.

The goal of this introductory chapter is primarily to give some historical perspective to the evolution of the subject. All detailed discussion is deferred to later chapters, where the topics are examined again in much greater depth. We start by reviewing the basic observed electrodynamic phenomena and their early

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<sup>1</sup>H. Kamerlingh Onnes, *Leiden Comm.* **120b**, **122b**, **124c** (1911).

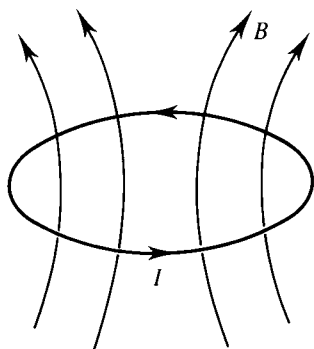
<sup>2</sup>G. Bednorz and K. A. Müller, *Z. Phys.* **B64**, 189 (1986).

phenomenological description by the Londons. We then briefly sketch the subsequent evolution of the concepts which are central to our present understanding. This quasi-historical review of the development of the subject is probably too terse to be fully understood on the first reading. Rather, it is intended to provide a quick overview to help orient the reader while reading subsequent chapters, in which the ideas are developed in sufficient detail to be self-contained. In fact, some readers have found this survey more useful to highlight the major points *after* working through the details in subsequent chapters.

## 1.1 THE BASIC PHENOMENA

What Kamerlingh Onnes observed was that the electrical resistance of various metals such as mercury, lead, and tin disappeared completely in a small temperature range at a critical temperature  $T_c$ , which is characteristic of the material. The complete disappearance of resistance is most sensitively demonstrated by experiments with persistent currents in superconducting rings, as shown schematically in Fig. 1.1. Once set up, such currents have been observed to flow without measurable decrease for a year, and a lower bound of some  $10^5$  years for their characteristic decay time has been established by using nuclear resonance to detect any slight decrease in the field produced by the circulating current. In fact, we shall see that under many circumstances we expect absolutely no change in field or current to occur in times less than  $10^{10}$  years! Thus, *perfect conductivity* is the first traditional hallmark of superconductivity. It is also the prerequisite for most potential applications, such as high-current transmission lines or high-field magnets.

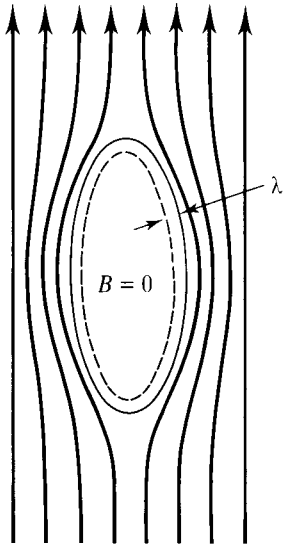
The next hallmark to be discovered was *perfect diamagnetism*, found in 1933 by Meissner and Ochsenfeld.<sup>3,4</sup> They found that not only a magnetic field is *excluded* from entering a superconductor (see Fig. 1.2), as might appear to be



**FIGURE 1.1**  
Schematic diagram of persistent current experiment.

<sup>3</sup>W. Meissner and R. Ochsenfeld, *Naturwissenschaften* **21**, 787 (1933).

<sup>4</sup>Actually, the diamagnetism is perfect only for *bulk* samples, since the field does penetrate a finite distance  $\lambda$ , typically approximately 500 Å.



**FIGURE 1.2**

Schematic diagram of exclusion of magnetic flux from interior of massive superconductor.  $\lambda$  is the penetration depth, typically only 500 Å.

explained by perfect conductivity, but also that a field in an originally normal sample is *expelled* as it is cooled through  $T_c$ . This certainly could *not* be explained by perfect conductivity, which would tend to trap flux *in*. The existence of such a reversible *Meissner effect* implies that superconductivity will be destroyed by a critical magnetic field  $H_c$ , which is related thermodynamically to the free-energy difference between the normal and superconducting states in zero field, the so-called condensation energy of the superconducting state. More precisely, this *thermodynamic critical field*  $H_c$  is determined by equating the energy  $H^2/8\pi$  per unit volume, associated with holding the field out against the magnetic pressure, with the condensation energy. That is,

$$\frac{H_c^2(T)}{8\pi} = f_n(T) - f_s(T) \quad (1.1)$$

where  $f_n$  and  $f_s$  are the Helmholtz free energies per unit volume in the respective phases in zero field. It was found empirically that  $H_c(T)$  is quite well approximated by a parabolic law

$$H_c(T) \approx H_c(0)[1 - (T/T_c)^2] \quad (1.2)$$

illustrated in Fig. 1.3. While the transition in zero field at  $T_c$  is of second order, the transition in the presence of a field is of first order since there is a discontinuous change in the thermodynamic state of the system and an associated latent heat.

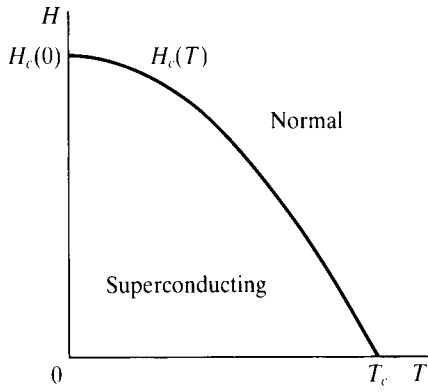


FIGURE 1.3  
Temperature dependence of the critical field.

## 1.2 THE LONDON EQUATIONS

These two basic electrodynamic properties, which give superconductivity its unique interest, were well described in 1935 by the brothers F. and H. London,<sup>5</sup> who proposed two equations to govern the microscopic electric and magnetic fields

$$\mathbf{E} = \frac{\partial}{\partial t} (\Lambda \mathbf{J}_s) \quad (1.3)$$

$$\mathbf{h} = -c \operatorname{curl} (\Lambda \mathbf{J}_s) \quad (1.4)$$

where

$$\Lambda = \frac{4\pi\lambda^2}{c^2} = \frac{m}{n_s e^2} \quad (1.5)$$

is a phenomenological parameter. It was expected that  $n_s$ , the *number density of superconducting electrons*, would vary continuously from zero at  $T_c$  to a limiting value of the order of  $n$ , the density of conduction electrons, at  $T \ll T_c$ . In (1.4), we introduce our notational convention of using  $\mathbf{h}$  to denote the value of the flux density on a microscopic scale, reserving  $\mathbf{B}$  to denote a macroscopic average value. Although notational symmetry would suggest using  $\mathbf{e}$  for the microscopic local value of  $\mathbf{E}$  in the same way, to avoid constant confusion with the charge  $e$  of the electron, we shall do so only in the few cases<sup>6</sup> where it is really useful. These notational conventions are discussed further in the appendix.

<sup>5</sup>F. and H. London, *Proc. Roy. Soc. (London)* **A149**, 71 (1935).

<sup>6</sup>The fundamental basis for our notational asymmetry in treating  $\mathbf{E}$  and  $\mathbf{B}$  is in the Maxwell equations  $\operatorname{curl} \mathbf{h} = 4\pi\mathbf{J}/c$  and  $\operatorname{curl} \mathbf{e} = -(1/c)\partial\mathbf{h}/\partial t$ . Superconductors in equilibrium can have nonzero  $\mathbf{J}_s$ , as described by the London equations, causing  $\mathbf{h}$  to vary on the scale of  $\lambda$ . But in equilibrium, or even steady state,  $\partial\mathbf{h}/\partial t = 0$ , so that  $\mathbf{e}$  is zero, or at least constant in space, so the use of both  $\mathbf{e}$  and  $\mathbf{E}$  offers no advantage. The distinction is useful only in discussing time-dependent phenomena such as motion of flux-bearing vortices in type II superconductors.

The first of these equations (1.3) describes perfect conductivity since any electric field *accelerates* the superconducting electrons rather than simply sustaining their velocity against resistance as described in Ohm's law in a normal conductor. The second London equation (1.4), when combined with the Maxwell equation  $\text{curl } \mathbf{h} = 4\pi\mathbf{J}/c$ , leads to

$$\nabla^2 \mathbf{h} = \frac{\mathbf{h}}{\lambda^2} \quad (1.6)$$

This implies that a magnetic field is exponentially screened from the interior of a sample with penetration depth  $\lambda$ , i.e., the Meissner effect. Thus, the parameter  $\lambda$  is operationally defined as a penetration depth; empirically, the temperature dependence of  $\lambda$  is found to be approximately described by

$$\lambda(T) \approx \lambda(0)[1 - (T/T_c)^4]^{-1/2} \quad (1.7)$$

The implications of the London equations are illustrated much more thoroughly in Chap. 2.

A simple, but unsound, "derivation" of (1.3) can be given by computing the response to a uniform electric field of a perfect normal conductor, i.e., a free-electron gas with mean free path  $\ell = \infty$ . In that case,  $d(m\mathbf{v})/dt = e\mathbf{E}$ , and since  $\mathbf{J} = nev$ , (1.3) follows. But this computation is not rigorous for the spatially nonuniform fields in the penetration depth, for which (1.3) and (1.4) are most useful. The fault is that the response of an electron gas to electric fields is non-local; i.e., the current at a point is determined by the electric field averaged over a region of radius  $\sim \ell$  about that point. Consequently, only fields that are uniform over a region of this size give a full response; in particular, the conductivity becomes *infinite* as  $\ell \rightarrow \infty$  *only* for fields filling all space. Since we are dealing here with an interface between a region with field and one with no field, it is clear that even for  $\ell = \infty$ , the effective conductivity would remain finite. For the case of a high-frequency current, this corresponds to the extreme anomalous limit of the normal skin effect, in which the surface resistance remains finite even as  $\ell \rightarrow \infty$ .

A more profound motivation for the London equations is the quantum one, emphasizing use of the vector potential  $\mathbf{A}$ , given by F. London<sup>7</sup> himself. Noting that the canonical momentum  $\mathbf{p}$  is  $(m\mathbf{v} + e\mathbf{A}/c)$ , and arguing that in the absence of an applied field we would expect the ground state to have zero net momentum (as shown in a theorem<sup>8</sup> of Bloch), we are led to the relation for the local average velocity in the presence of the field

$$\langle \mathbf{v}_s \rangle = \frac{-e\mathbf{A}}{mc}$$

<sup>7</sup>F. London, *Superfluids*, vol. I, Wiley, New York, 1950.

<sup>8</sup>This theorem is apparently unpublished, though famous. See p. 143 of the preceding reference.



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