Financial Modelling in Python

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WILEY
A John Wiley and Sons, Ltd., Publication

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This edition firs published 2009

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John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom

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Library of Congress Cataloging-in-Publication Data
Fletcher, Shayne.
Financial modeling in Python / Shayne Fletcher and Christopher Gardner.
p. cm. — (Wiley financ series)
Includes bibliographical references and index.
ISBN 978-0-470-98784-1 (cloth : alk. paper) 1. Finance—Mathematical models—Computer programs.
2. Python (Computer program language) I. Gardner, Christopher. II. Title.
HG106.F59 2009
332.0285′5133—dc22
2009019336

ISBN 978-0-470-98784-1

A catalogue record for this book is available from the British Library.

Typeset in 10/12pt Times by Aptara Inc., New Delhi, India
Printed in Great Britain by Antony Rowe Ltd, Chippenham, Wiltshire

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1

Welcome to Python

In this introductory chapter, we welcome the reader to Python and make some arguments that we hope will serve to motivate Python programming in finance

1.1 WHY PYTHON?

We contend that the Python programming language is particularly suited to quantitative analysts/programmers working in the fiel of financia engineering. This assertion centres on two axes: the first Python's expressiveness and high-level nature; the second, Python's extensibility and interoperability with other programming languages. Other (arguably not so,) minor arguments to be made for Python programming in general, are the benefit to be had from the use of Python's wealth of standard libraries ('Python comes with batteries included') and Python's support for functional programming idioms.

We certainly do not wish to assert that Python is 'better' in any way than other programming languages (we rejoice in the diversity of programming languages!), but instead wish to emphasise how Python can interoperate with and complement other languages to be found in financia institutions.

1.1.1 Python is a General-Purpose High-Level Programming Language

Python's high-level nature and its rich collection of built-in data types serve to allow the analyst/programmer to focus more on the problems they are solving and less on low-level mechanical constructs relating to such things as memory management in contrast to other programming languages in common use in this domain. Taken together with the simplicity and renowned expressiveness of the Python programming language syntax, this goes some way to explaining the often reported large productivity pickups that result from choosing Python over other languages. As another consequence of these features, programs in Python can be expected to be much shorter and more concise than their representations in other programming languages.

For quantitative analysts, and indeed computational scientists in general, very useful Python packages exist to make the task of numerical analysis programs much easier (SciPy).[1] In addition, quantitative analysts 'in the field well know that writing programs for financ will often typically involve much more than numerical code alone, as many of these programs are concerned with acquiring and organising data on which the numerical aspects of the program are applied. We have often found that these tasks can be achieved in less lines of code and with significantl less effort in Python than other programming languages.

[1] SciPy is open-source (Python) software for mathematics, science and engineering. See http://www.scipy.org for details for example.

1.1.2 Python Integrates Well with Data Analysis, Visualisation and GUI Toolkits

Another compelling argument for the use of Python by quantitative analysts is the ease with which Python integrates with visualisation software such as GNUPlot[2] making it possible for the analyst to construct personalised 'Matlab-like'[3] enivronments. Furthermore, quantitative analysts generally have neither the interest or time to invest in producing graphical user interfaces (GUIs). They can be nonetheless important. Python provides Tk-based[4] GUI tools making it straightforward to wrap programs into GUIs. Readers interested in learning more about how Python can be integrated with GUI building, data analysis and visualisation software are particularly recommended to consult Hans Peter Langtangen's *Python Scripting for Computational Science* [14].

1.1.3 Python 'Plays Well with Others'

A variety of techniques exist to extend Python from the C and C++ programming languages. Conversely, a Python interpreter is easily embedded in C and C++ programs. In the world of financia engineering, C/C++ prevails and large bodies of this code exist in most financia institutions. The ability for new programs to be written in Python that can interoperate with these code investments is a huge victory for the analyst and the institutions considering its use.

1.2 COMMON MISCONCEPTIONS ABOUT PYTHON

There are a number of ill-informed arguments oft encountered that, when made, impede the propogation or acceptance of Python programming in finance The most common include 'it is not fast enough', 'it does not engender a clear structure to your code' and (the most incorrect proposition) 'it has no type checking'. In fact, for most applications Python is 'fast enough' and those parts of the application that are computationally intensive can be implemented in fast 'traditional' programming languages like C or C++, bringing the best of both worlds. As for the argument that Python does not engender a clear structure to code, this is hard to understand. Python supports encapsulation at the function, class and namespace levels as well as any of the modern object-oriented or multiparadigm programming languages. Now, what about Python having no type checking? This is simply wrong. Python is dynamically typed, that is to say, type checking is performed at run-time but type checking does happen! Furthermore, the absence of explicit type declarations in the code is one of the keys to why a Python program can be so much more succinct and faster to produce than languages with static type checking. Staying with the topic of Python's type system, it is interesting to note that Python's dynamic type system implicitly supports generic programming. Consider an example taken from the `ppf.math`[5] module

```
def solve_tridiagonal_system(N, a, b, c, r):
  ...
  return result
```

[2] GNUPlot is a cross platform function plotting utility. See http://www.gnuplot.info for details.

[3] Matlab is a numerical computing environment and programming language popular in both industry and academia. See http://www.mathworks.com/ for details.

[4] Tk is an open-source, cross-platform graphical user interface toolkit. See http://www.tcl.tk for details.

[5] Look ahead to the section 'Roadmap for this book' for an explanation of PPF.

Here `N` is the dimension of an $N \times N$ linear system, `a, b, c` are the subdiagonal, diagonal, and superdiagonal of the system respectively, and `r` the right hand side. The point to be made is that the function will work with any types that are consistent with being *Indexable* (i.e. satisfy an *Indexable* concept in the C++0x[6] sense of the word). This admits the use of the function with Python lists, NumPy[7] arrays or some other user-define array type ... generic programming!

1.3 ROADMAP FOR THIS BOOK

Chapter-by-chapter this book gradually presents a practical body of working code referred to as PPF or the `ppf` package, that implements a minimal but extensible Python-based financia engineering system.

Chapter 2 looks at the overall topology of the `ppf` package, its dependencies and how to build, install and test it (newcomers to Python may be served by looking ahead to Appendix A where a quick tutorial on Python basics is offered).

Chapter 3 considers the topic of implementing Python extension modules in C++ with an emphasis on fostering interoperability with existing C++ financia engineering systems and, in particular, how certain functionality present in `ppf` in fact is underlied by C++ in this fashion.

Chapter 4 lays the groundwork for later chapters (concerned with pricing using techniques from numerical analysis) in that it presents those mathematical algorithms and tools that arise over and over again in computational quantitative analysis, including:

(1) (pseudo) random number generation;
(2) estimation of the standard normal cumulative distribution function;
(3) a variety of interpolation schemes;
(4) root-findin algorithms;
(5) various operations for linear algebra;
(6) generalised linear least-squares data fitting
(7) stable calculation techniques for computing quadratic and cubic roots; and
(8) calculation of the expectation of a function of a random variable.

Chapter 5 looks at how the `ppf` represents common market information such as discount-factor functions and volatility surfaces.

Chapter 6 is entirely concerned with looking at the data structures used in the `ppf` for representing financia structures: ‘fl ws’, ‘legs’, ‘exercise opportunities’, ‘trades’ and the like.

Chapter 7 details the concepts and classes that govern the interactions between the trade representations and pricing models in the `ppf` package.

Chapter 8 offers an implementation of a fully functional Hull–White model in Python, where the characteristic features of the model are assembled from (in as much as is possible) functionally orthogonal components.

Chapter 9 present two general numerical pricing frameworks invariant over pricing models: one lattice based, the other Monte-Carlo based.

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[6] The next version of the C++ standard, expected to be completed in 2009.

[7] The fundamental package for scientifi computing with Python. SciPy (as indeed PPF) depends on NumPy. See http://numpy.scipy.org for details.

Chapter 10 applies the pricing frameworks and the Hull–White model developed in the preceding chapters to pricing financia structures, specificall , Bermudan swaptions and target redemption notes.

Chapter 11, while keeping things tractable, introduces the idea of and practical techniques for C++/Python 'Hybrid Systems' against the backdrop of existing derivative security pricing and risk management systems in C++.

Chapter 12 gives concrete examples of implementing COM servers in Python and utilising the functionality so exposed in the context of Microsoft Excel.

In the appendices section, Appendix A offers newcomers to Python a brief tutorial. Appendix B provides a primer for the use of the C++ Boost.Python library for fostering interoperability between C++ and Python. Appendix C covers the mathematics of the Hull–White model and Appendix D the mathematics of a simple regression scheme for determining the early exercise premium of a callable structure when pricing using Monte-Carlo techniques.

2 The PPF Package

The source code accompanying this book implements a minimal library, `ppf`, for exploring financia modelling in Python. The sections ahead outline the structure and ideas of the package.

The following is a firs example of a financia program expressed in Python – the 'Hello World' of Quantitative Analysis programs, that is, the Black–Scholes formula for a European option on a single asset:

```
from math import log, sqrt, exp
from ppf.math import N

def black_scholes(S, K, r, sig, T, CP, *arguments, **keywords):
  """The classic Black and Scholes formula.

  >>> print black_scholes(S=42., K=40., r=0.1, sig= 0.2, T=0.5,
  CP=CALL) 4.75942193531

  >>> print black_scholes(S=42., K=40., r=0.1, sig= 0.2, T=0.5,
  CP=PUT) 0.808598915338

  """
  d1 = (log(S/K) + (r + 0.5*(sig*sig))*T)/(sig*sqrt(T))
  d2 = d1 - sig*sqrt(T)

  return CP*S*N(CP*d1) - CP*K*exp(-r*T)*N(CP*d2)

CALL, PUT = (1, -1)

def _test():
  import doctest
  doctest.testmod()

if __name__ == '__main__': _test()
```

2.1 PPF TOPOLOGY

The `ppf` library is a Python package containing a family of sub-packages. The `black_scholes` function listed above is housed in the `ppf.core` subpackage. The topology of `ppf` is as follows:

```
ppf/
  com/
  core/
  date_time/
```

```
    market/
    math/
    model/
      hull_white/
        lattice/
        monte_carlo/
    pricer/
      payoffs/
    test/
    utility/
```

Here is a brief summary of the nature and main roles of each of the ppf sub-packages:

com	COM servers wrapping ppf market, trade and pricing functionality (see Chapter 12).
core	Types and functions relating to the representation of financia quantities such as fl ws and LIBOR rates.
date_time	Date and time manipulation and computations.
market	Types and functions for the representation of common curves and surfaces that arise in financia programming such as discount factor curves and volatility surfaces.
math	General mathematical algorithms.
model	Code specifi to implementing numerical pricing models.
pricer	Types and functions for the purpose of valuing financia structures.
text	The ppf unit test suite.
utility	Utilities of a less numerical, general nature such as algorithms for searching and sorting.

2.2 UNIT TESTING

Code in the ppf library employs two approaches to testing: interactive Python session testing using the doctest module and formalised unit testing using the PyUnit module. Both of these testing frameworks are part of the Python standard libraries.

2.2.1 doctest

The way that the doctest module works is to search a module for pieces of text that look like interactive Python sessions, and then to execute those sessions to verify that they work as expected. In this way ppf modules come with a form of tutorial-like executable documentation:

```
C:\Python25\lib\site-packages\ppf\core>python black_scholes.py -v
python black_scholes.py -v
Trying:
    print black_scholes(S=42., K=40., r=0.1, sig= 0.2, T=0.5, CP=CALL)
Expecting:
    4.75942193531
ok
```

```
Trying:
    print black_scholes(S=42., K=40., r=0.1, sig= 0.2, T=0.5, CP=PUT)
Expecting:
    0.808598915338
ok
2 items had no tests:
    __main__
    __main__._test
1 items passed all tests:
   2 tests in __main__.black_scholes
2 tests in 3 items.
2 passed and 0 failed.
Test passed.
```

2.2.2 PyUnit

A full suite of unit tests for all modules in the `ppf` package is provided in the `ppf.test` sub-package. The tests can be run module-by-module or, to execute all tests in one go, a driver 'test_all.py' is provided:

```
C:\Python25\Lib\site-packages\ppf\test>python test_all.py --verbose
python test_all.py --verbose
test_call (test_core.black_scholes_tests) ... ok
test_put (test_core.black_scholes_tests) ... ok
test (test_core.libor_rate_tests) ... ok
    .
    .
    .
test_upper_bound (test_utility.bound_tests) ... ok
test_equal_range (test_utility.bound_tests) ... ok
test_bound (test_utility.bound_tests) ... ok
test_bound_ci (test_utility.bound_tests) ... ok

----------------------------------------------------------------------
Ran 51 tests in 25.375s

OK
```

2.3 BUILDING AND INSTALLING PPF

In this section we look at what it takes to build and install the `ppf` package.

2.3.1 Prerequisites and Dependencies

`ppf` is composed of a mixture of pure Python modules underlied by some supporting extension modules implemented in standard C++. Accordingly, to build and install `ppf` requires a modern C++ compiler. The C++ extension modules have some library dependencies of their own, notably the Boost C++ libraries and the Blitz++ C++ library. Instructions for downloading

and installing the Boost C++ libraries can be found at `http://www.boost.org` and instructions for Blitz++ can be found at `http://www.oonumerics.org`. Naturally, an installation of Python is also required. On Windows, the authors favour the freely available ActiveState Python distribution, see `http://www.activestate.com` for download and installation details. Also required on the Python side for `ppf` is an installation of the NumPy package, see `http://www.scipy.org` for download and installation details.

2.3.2 Building the C++ Extension Modules

The `ppf` C++ extension modules are most conveniently built using the Boost.Build system[1] a copy of which is included with the `ppf` sources. Also provided with the `ppf` sources for the convenience of Windows users is a pre-built executable 'bjam.exe'. Although these notes will become a little Windows-centric at this point, the basic principles will hold for *NIX users also. On Windows, the `ppf` package has been successfully built and tested with the Microsoft Visual Studio C++ compiler versions 7.1, 8.0 (express edition), 9.0 (express edition), mingw/gcc-3.4.5,[2] mingw/gcc-4.3.0 with Python versions 2.4 and 2.5, Boost versions 1.33.1, 1.34.0, 1.35, 1.36, 1.37 and Blitz++ version 0.9. The `ppf` package has also been built and tested on the popular Linux-based operating system, Ubuntu-8.04.1 with Boost version 1.36.0, Blitz++ version 0.9 and gcc-4.2.3.

In the remainder of this section, without loss of generality, we will assume a Windows operating system, Blitz++ version 0.9, the ActiveState distribution of Python version 2.5 and Boost version 1.36.

Build Instructions

- Prerequisites

```
- Copy c:/path/to/ppf/ext/bjam.exe to somewhere in your %PATH%
- Install
  o Blitz++-0.9
  o Boost-1.36
  o ActiveState Python 2.5
  o NumPy for Python 2.5 (version 1.0.4 or 1.1.0)
- Edit as appropriate for your site
  o c:/path/to/ppf/ext/build/user-config.jam
  o c:/path/to/ppf/ext/build/site-config.jam
```

- Build

```
- c:/path/to/ppf>cd ext&&bjam [debug|release]
 This will create:
 o c:/path/to/ppf/ppf/math/ppf_math.pyd and
 o c:/path/to/ppf/ppf/date_time/ppf_date_time.pyd
```

[1] See http://www.boost.org/doc/tools/build/index.html.
[2] Minimalist GNU for Windows – see http://www.mingw.org.

2.3.3 Installing the PPF Package

Assuming the steps of the previous section have been performed, installation of the `ppf` package which relies on the standard Python Distutils package is very simple.

- Install
  - `c:/path/to/ppf>python setup.py install`

which will copy the `ppf` package to the standard Python installation location (`c:/python25/lib/site-packages/ppf`).

2.3.4 Testing a PPF Installation

The easiest way to verify a `ppf` installation is to run the `ppf` unit test suite.

- Test
  - `c:/python25/lib/site-packages/ppf/test>python test_all.py --verbose`

3
Extending Python from C++

It is usual in financia institutions that make use of quantitative analysis programs to have a considerable investment in C++. Thus it can be important to foster interoperability between C++ and Python. This chapter studies how Python modules can be implemented in C++ by means of the Boost.Python[1] library (see also Appendix B for a primer on the Boost.Python library).

3.1 BOOST.DATE_TIME TYPES

It is common in quantitative analysis programming to require manipulation of and computations involving dates. The 'Python Library' contains excellent functionality for such activities. Pricing systems written in C++, however, will be implemented using C++ datatypes for the representation of dates and times. For pricing frameworks implemented in a hybrid of Python and C++, it would be convenient to settle on a common representation of these fundamental types. Accordingly, in this section we demonstrate the 'reflection of functionality from the C++ Boost.Date_Time library to Python.

Our reflectio of the C++ date types into Python will be housed in the Python module 'ppf_date_time.pyd', implemented in C++. We declare this intention in the entry point to our Python module in the fil 'module.cpp':

```
#include <boost/python/module.hpp>

namespace ppf
{
  namespace date_time
  {
    void register_date();
    void register_date_more();

  } // namespace date_time

} // namespace ppf

BOOST_PYTHON_MODULE(ppf_date_time)
{
  using namespace ppf::date_time;

  register_date();
  register_date_more();
}
```

[1] Boost provides free peer-reviewed portable C++ source libraries. See http://www.boost.org for details.

In 'register_date.cpp' we instantiate Boost.Python `class_` objects describing the C++ types and functions we intend to use from Python:

```
void register_date()
{
  using namespace boost::python;
  namespace bg = boost::gregorian;
  namespace bd = boost::date_time;

  // types and functions ...

  class_<bg::date>(
      "date"
     ,"A date type based on the gregorian calendar"
     , init<>("Default construct not_a_date_time"))
    .def(init<bg::date const&>())
    .def(init
         <
           bg::greg_year
         , bg::greg_month
         , bg::greg_day
         >((arg("y"), arg("m"), arg("d"))
       , "Main constructor with year, month, day "))
    .def("year", &bg::date::year)
    .def("month", &bg::date::month)
    .def("day", &bg::date::day)

    // ...

    ;

  class_<std::vector<bg::date> >(
      "date_vec"
    , "vector (C++ std::vector<date> ) of date")
    .def(vector_indexing_suite<std::vector<bg::date> >())
    ;

  // more types and functions ...
}
```

Once exposed in this fashion, the types so define in the `ppf_date_time` module are imported into the `ppf` subpackage `ppf.date_time` by means of import statements in the module's '__init__.py':

```
from ppf_date_time import *
```

3.1.1 Examples

IMM Dates

As an example of what we have achieved, let's see how, in Python, we can compute so-called IMM (international money market) dates for a given year, i.e. the 3rd Wednesday of March, June, September, and December in the year. The `ppf.date_time` package provides the

module `nth_imm_of_year` in which is define `class nth_imm_of_year`. The workhorse of the class implementation is the Boost.Date_Time function `nth_kday_of_month`:

```
from ppf_date_time import  \
     weekdays              \
   , months_of_year        \
   , nth_kday_of_month     \
   , year_based_generator

class nth_imm_of_year(year_based_generator):
  '''Calculate the nth IMM date for a given year

  '''
  first = months_of_year.Mar
  second = months_of_year.Jun
  third = months_of_year.Sep
  fourth = months_of_year.Dec

  def __init__(self, which):
    year_based_generator.__init__(self)
    self._month = which

  def get_date(self, year):
    return nth_kday_of_month(
          nth_kday_of_month.third
        , weekdays.Wednesday
        , self._month).get_date(year)

  def to_string(self):
    pass
```

Exercising the `class nth_imm_of_year` functionality in an interactive Python session goes like this:

```
>>> from ppf.date_time import *
>>> imm = nth_imm_of_year
>>> imm_dates = []
>>> imm_dates.append(imm(imm.first).get_date(2005))
>>> imm_dates.append(imm(imm.second).get_date(2005))
>>> imm_dates.append(imm(imm.third).get_date(2005))
>>> imm_dates.append(imm(imm.fourth).get_date(2005))
>>> for t in imm_dates:
...   print t
2005-Mar-16
2005-Jun-15
2005-Sep-21
2005-Dec-21
```

With `class nth_imm_of_year` some useful questions regarding IMM dates can now be answered elegantly and easily. For example, what is the IMM date immediately preceding a given date? This is answered in the `ppf.date_time.first_imm_before` module:

```
from ppf_date_time import  \
     weekdays              \
```

```
   , months_of_year        \
   , nth_kday_of_month     \
   , year_based_generator
from nth_imm_of_year import *

def first_imm_before(start):
  '''Find the IMM date immediately preceding the given date.
  '''
  imm = nth_imm_of_year
  first_imm_of_year = imm(imm.first).get_date(start.year())
  imm_date = None
  if start <= first_imm_of_year:
    imm_date = imm(imm.fourth).get_date(start.year() - 1)
  else:
    for imm_no in reversed([imm.first, imm.second, imm.third,
                            imm.fourth]):
      imm_date = imm(imm_no).get_date(start.year())
      if imm_date < start:
        break

  return imm_date
```

In an interactive Python session:

```
>>> from ppf.date_time import *
>>> print first_imm_before(date(2007, Jun, 27))
2007-Jun-20
```

The `ppf.date_time` package also contains the symmetric `first_imm_after` function.

Holidays, Rolls and Year Fractions

Other common activities in financia modelling include determining if a date is a business day, 'rolling' a date to a business day and the computation of elapsed time between two dates according to common market conventions.

The `ppf.date_time.shift_convention` module shows an easy way to emulate C++ enum types:

```
class shift_convention:
    none                 \
  , following            \
  , modified_following   \
  , preceding            \
  , modified_preceding = range(5)
```

This idiom is employed again in the `ppf.date_time.day_count_basis` module:

```
class day_count_basis:
    basis_30360   \
  , basis_act_360 \
  , basis_act_365 \
  , basis_act_act = range(4)
```

The ppf.date_time.is_business_day module provides the means to answer the question of whether or not a given date is a business day:

```
from ppf_date_time import weekdays

def is_business_day(t, financial_centres=None):
  ''' Test whether the given date is a business day.
      In this version, only weekends are considered
      holidays.
  '''
  Saturday, Sunday = weekdays.Saturday, weekdays.Sunday
  return t.day_of_week().as_number() != Saturday \
       and t.day_of_week().as_number() != Sunday
```

The ppf.date_time.shift module provides functionality to 'shift' a date according to the common market shift conventions:

```
from ppf_date_time import *
from is_business_day import *
from shift_convention import *

def shift(t, method, holiday_centres=None):
  d = date(t)
  if not is_business_day(d):
    if method == shift_convention.following:
      while not is_business_day(d, holiday_centres):
        d = d + days(1)
    elif method == shift_convention.modified_following:
      while not is_business_day(d, holiday_centres):
        d = d + days(1)
      if d.month().as_number() != t.month().as_number():
          d = date(t)
          while not is_business_day(d, holiday_centres):
            d = d - days(1)
    elif method == shift_convention.preceding:
      while not is_business_day(d, holiday_centres):
        d = d - days(1)
    elif method == shift_convention.modified_preceding:
      while not is_business_day(d, holiday_centres):
        d = d - days(1)
      if d.month().as_number() != t.month().as_number():
        while not is_business_day(d, holiday_centres):
          d = d + days(1)
    else: raise RuntimeError, "Unsupported method"

  return d
```

The ppf.date_time.year_fraction module provides functionality to compute year fractions:

```
from ppf_date_time \
     import date, gregorian_calendar_base
from day_count_basis import *
```

```
is_leap_year = gregorian_calendar_base.is_leap_year

def year_fraction(start, until, basis):
  '''Compute accruals
  '''
  result = 0
  if basis == day_count_basis.basis_act_360:
    result = (until - start).days()/360.0
  elif basis == day_count_basis.basis_act_365:
    result = (until - start).days()/365.0
  elif basis == day_count_basis.basis_act_act:
    if start.year() != until.year():
      start_of_to_year = date(until.year(), 1, 1)
      end_of_start_year = date(start.year(), 12, 31)
      result = (end_of_start_year - start).days()/ \
          (365.0, 366.0)[is_leap_year(start.year())] \
        +  (int(until.year()) - int(start.year()) - 1) + \
           (until - start_of_to_year).days()/ \
              (365.0, 366.0)[is_leap_year(until.year())]
    else:
      result = (until - start).days()/ \
                (365.0, 366.0)[is_leap_year(util.year())]
  elif basis == day_count_basis.basis_30360:
    d1, d2 = start.day(), until.day()
    if d1 == 31:
        d1 -= 1
    if d2 == 31:
        d2 -= 1
    result = (int(d2) - int(d1)) + \
             30.0*(int(until.month()) - int(start.month())) + \
                      360.0*(int(until.year()) - int(start.year()))
    result = result / 360.0
  else:
    raise RuntimeError, "Unsupported basis"

  return result
```

In the following interactive session, the year fraction between two dates is computed under a variety of different day count basis conventions:

```
>>> from ppf.date_time import *
>>> add_months = month_functor
>>> Nov = months_of_year.Nov
>>> begin = date(2004, Nov, 21)
>>> until = begin + add_months(6).get_offset(begin)
>>> year_fraction(begin, until, day_count_basis.basis_30360)
0.5
>>> year_fraction(begin, until, day_count_basis.basis_act_365)
0.49589041095890413
>>> year_fraction(begin, until, day_count_basis.basis_act_act)
0.49285126132195523
```

3.2 BOOST.MULTIARRAY AND SPECIAL FUNCTIONS

The use of multidimensional arrays in quantitative analysis programs is ubiquitous. Python, or rather the Python libraries provide a variety of types that serve for their representation. Like the date types of the previous section, however, we prefer to emphasise interoperability with C++ and so, to this end, might favour reflectio of C++ array types into Python. The `ppf` package exposes the Boost.MultiArray multidimensional array types `boost::multi_array<double,N>` for `N` = 1, 2, 3 to Python. To achieve this, advantage was taken of a C++ template meta-program that facilitates reflectio of the arrays, the code for which is present in the source code accompanying this book (see 'ext/boost/multi_array/multi_array hpp').

The array types are housed in the `ppf_math` module implemented in the C++ Python extension 'ppf_math.pyd' and imported into the namespace of the `ppf.math` subpackage. Usage of the array types is natural and intuitive. Here is an example taken from the `ppf.math` unit tests:

```
class solve_upper_diagonal_system_tests(unittest.TestCase):
  def test(self):

  # Solve upper diagonal system of linear equations ax = b
  # where
  #
  # a = 3x3
  #     [  1.75    1.5    -2.5
  #          0     -0.5    0.65
  #          0        0    0.25 ]
  #
  # and b = [0.5, -1.0, 3.5].

  a = ppf.math.array2d([3,3])
  a[0, 0], a[0, 1], a[0, 2] = (1.75, 1.5, -2.5)
  a[1, 0], a[1, 1], a[1, 2] = (0.0, -0.5,  0.65)
  a[2, 0], a[2, 1], a[2, 2] = (0.0,  0.0,  0.25)
  b = ppf.math.array1d([3])
  b[0] =  0.5
  b[1] = -1.0
  b[2] =  3.5

  # Expected solution vector is x = [2.97142857  20.2  14.0].

  x = ppf.math.solve_upper_diagonal_system(a, b)
  assert len(x) == 3 and math.fabs(x[0] - 2.971428571) < 1.0e-6 \
         and math.fabs(x[1] - 20.2) < 1.0e-6 and math.fabs(x[2] -
         14.0) < 1.0e-6
```

In addition to the multi-array types, the module `ppf_math` also exposes some useful utility functions implemented in C++. In the fil 'ppf/math/limits hpp' are the following template function definitions

```
#if !defined(LIMITS_5DDE828B_9989_44F5_9728_47AA72323D96_INCLUDED)
#  define LIMITS_5DDE828B_9989_44F5_9728_47AA72323D96_INCLUDED
```

```
#  if defined(_MSC_VER) && (_MSC_VER >= 1020)
#    pragma once
#  endif // defined(_MSC_VER) && (_MSC_VER >= 1020)

#  include <boost/config.hpp>

#  include <limits>

namespace ppf { namespace math {

template <class T>
T epsilon()
{
  return std::numeric_limits<T>::epsilon();
}

template <class T>
T min BOOST_PREVENT_MACRO_SUBSTITUTION ()
{
  return (std::numeric_limits<T>::min)();
}

template <class T>
T max BOOST_PREVENT_MACRO_SUBSTITUTION ()
{
  return (std::numeric_limits<T>::max)();
}

}} // namespace ppf::math

#endif//!defined(LIMITS_5DDE828B_9989_44F5_9728_47AA72323D96_INCLUDED)
```

In 'ext/lib/math/src/register_special_functions.cpp', instantiations of these templates are exposed to Python:

```
#include <boost/python/def.hpp>

#include <ppf/math/limits.hpp>

namespace ppf { namespace math {

void register_special_functions()
{
  using namespace boost::python;

  def("epsilon", epsilon<double>);
  def("min_flt", min BOOST_PREVENT_MACRO_SUBSTITUTION <double>);
  def("max_flt", max BOOST_PREVENT_MACRO_SUBSTITUTION <double>);
}

}} // namespace ppf::math
```

An example of the use of the `epsilon` function is again provided by a `ppf.math` unit test:

```
class bisect_tests(unittest.TestCase):
  def test1(self):
    tol = 5*ppf.math.epsilon()
    left, right, num_its = \
          ppf.math.bisect(lambda x: x*x + 2.0*x - 1.0
                          , -3, -2
                          , lambda x, y: math.fabs(x-y) < tol, 100)
```

Further examples of the use of these special functions can be found in the next chapter.

3.3 NUMPY ARRAYS

Despite the efforts of the preceding section regarding reflectio of C++ Boost.MultiArray types into Python, in practice, when working in Python, the authors have found the facilities of NumPy arrays to be far more convenient (NumPy was mentioned briefl in section 1.2). Specificall , their notational conveniences and the large body of functionality provided by the NumPy library motivates their use in Python beyond the argument of C++ interoperability. Indeed, when working in C++, a library dedicated to scientifi manipulation of arrays such as Blitz++[2] wins the authors' favour for such work over 'lower-level' container types like native C arrays or Boost.MultiArray types. But now to the crux of the matter. If we haven't made this point earlier then we'll make it for the firs time now. One of the great strengths of Python is the ability to drop into C or C++ code 'when performance counts'. That is, the ability to factor out that characteristic operation that must be done as efficientl as possible and pull it down into a compiled component is essential. Now, in the fiel of numerical programming, doesn't that characteristic operation almost always involve operating on arrays of data?

So, can we have it all? Can we have the convenience of NumPy in Python combined with the convenience and efficien y of Blitz++ in C++ where the data is shared between these array types? The short answer is 'yes we can', as we will demonstrate in the next subsection.

3.3.1 Accessing Array Data in C++

This subsection is concerned with the topic of accessing a NumPy array's data in C++. To do this, we need to work with the Python C API and we'll also take advantage of Boost.Python where we can. The approach is fairly idiomatic and can be more or less wrapped up in a set of reasonably small utility functions. Let's begin with this most simple of functions from 'ppf/util/python/detail/decref.hpp':

```
#if !defined(DECREF_4A1F1D9D_CE18_4CA1_AF52_DA1C51847FB4_INCLUDED)
#  define DECREF_4A1F1D9D_CE18_4CA1_AF52_DA1C51847FB4_INCLUDED

#  if defined(_MSC_VER) && (_MSC_VER >= 1020)
#    pragma once
#  endif // defined(_MSC_VER) && (_MSC_VER >= 1020)
```

[2] Blitz++ is a C++ class library for scientifi computing which provides performance on par with Fortran 77/90. See http://www.oonumerics.org/blitz for details.