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Sandra Luna McCune, Ph.D.

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By Sandra Luna McCune, Ph.D.

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***About the Author***

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***Dedication***

This book is dedicated to my grandchildren—Richard, Rose, Jude, Sophia, Josephine, and Myla Mae. They fill my life with joy!

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# Introduction

This book is organized around the Grade 8 Common Core State Standards for Mathematics. These standards define what eighth-grade students are expected to understand and be able to do in their study of mathematics. They include content standards and mathematical practice standards.

In Grade 8, the content standards are grouped under five domains:

- The Number System
- Expressions and Equations
- Functions
- Geometry
- Statistics and Probability

## The Number System

### Know that There Are Numbers that Are Not Rational, and Approximate Them by Rational Numbers

- **CCSS.Math.Content.8.NS.A.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers, show that the decimal expansion repeats eventually, and convert a decimal expansion that repeats eventually into a rational number.
- **CCSS.Math.Content.8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi^2$). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

## Expressions and Equations

### Work with Radicals and Integer Exponents

- **CCSS.Math.Content.8.EE.A.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.

- **CCSS.Math.Content.8.EE.A.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
- **CCSS.Math.Content.8.EE.A.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times* $10^8$ *and the population of the world as 7 times* $10^9$*, and determine that the world population is more than 20 times larger than the U.S. population.*
- **CCSS.Math.Content.8.EE.A.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

## Understand the Connections Between Proportional Relationships, Lines, and Linear Equations

- **CCSS.Math.Content.8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
- **CCSS.Math.Content.8.EE.B.6** Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a nonvertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$.

## Analyze and Solve Linear Equations and Pairs of Simultaneous Linear Equations

- **CCSS.Math.Content.8.EE.C.7** Solve linear equations in one variable.
  - **CCSS.Math.Content.8.EE.C.7.A** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).
  - **CCSS.Math.Content.8.EE.C.7.B** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

- **CCSS.Math.Content.8.EE.C.8** Analyze and solve pairs of simultaneous linear equations.

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  - **CCSS.Math.Content.8.EE.C.8.A** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
  - **CCSS.Math.Content.8.EE.C.8.B** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
  - **CCSS.Math.Content.8.EE.C.8.C** Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

# Functions

## Define, Evaluate, and Compare Functions

- **CCSS.Math.Content.8.F.A.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- **CCSS.Math.Content.8.F.A.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
- **CCSS.Math.Content.8.F.A.3** Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4), and (3, 9), which are not on a straight line.*

## Use Functions to Model Relationships Between Quantities

- **CCSS.Math.Content.8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two $(x, y)$ values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- **CCSS.Math.Content.8.F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

# Geometry

## Understand Congruence and Similarity Using Physical Models, Transparencies, or Geometry Software

- **CCSS.Math.Content.8.G.A.1** Verify experimentally the properties of rotations, reflections, and translations:
  - **CCSS.Math.Content.8.G.A.1.A** Lines are taken to lines, and line segments to line segments of the same length.
  - **CCSS.Math.Content.8.G.A.1.B** Angles are taken to angles of the same measure.
  - **CCSS.Math.Content.8.G.A.1.C** Parallel lines are taken to parallel lines.
- **CCSS.Math.Content.8.G.A.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- **CCSS.Math.Content.8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- **CCSS.Math.Content.8.G.A.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- **CCSS.Math.Content.8.G.A.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so*.

## Understand and Apply the Pythagorean Theorem

- **CCSS.Math.Content.8.G.B.6** Explain a proof of the Pythagorean theorem and its converse.
- **CCSS.Math.Content.8.G.B.7** Apply the Pythagorean theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
- **CCSS.Math.Content.8.G.B.8** Apply the Pythagorean theorem to find the distance between two points in a coordinate system.

## Solve Real-World and Mathematical Problems Involving Volume of Cylinders, Cones, and Spheres

- **CCSS.Math.Content.8.G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

# Statistics and Probability

## Investigate Patterns of Association in Bivariate Data

- **CCSS.Math.Content.8.SP.A.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- **CCSS.Math.Content.8.SP.A.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
- **CCSS.Math.Content.8.SP.A.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height*.
- **CCSS.Math.Content.8.SP.A.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

# Mathematical Practice

- **CCSS.Math.Practice.MP.1** Make sense of problems and persevere in solving them.
- **CCSS.Math.Practice.MP.2** Reason abstractly and quantitatively.
- **CCSS.Math.Practice.MP.3** Construct viable arguments and critique the reasoning of others.
- **CCSS.Math.Practice.MP.4** Model with mathematics.
- **CCSS.Math.Practice.MP.5** Use appropriate tools strategically.
- **CCSS.Math.Practice.MP.6** Attend to precision.
- **CCSS.Math.Practice.MP.7** Look for and make use of structure.
- **CCSS.Math.Practice.MP.8** Look for and express regularity in repeated reasoning.

# 1. The Number System

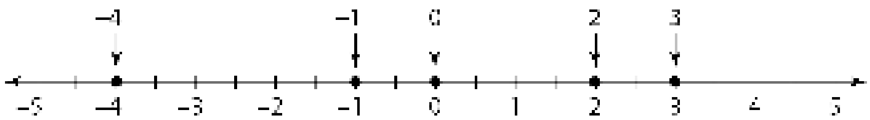
In this chapter, you will extend your understanding of the rational numbers, learn about irrational numbers, and recognize the real numbers as the set consisting of all the rational and irrational numbers. You will use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a real number line, and approximate the value of irrational expressions.

## Understanding Rational Numbers

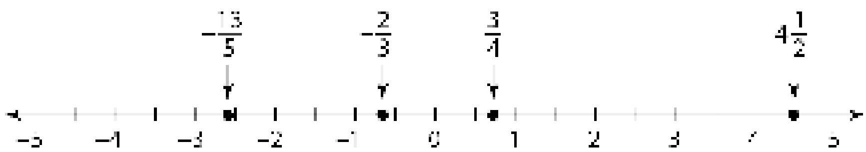
### (CCSS.Math.Content.8.NS.A.1)

A **rational number** is any number that can be expressed as $\frac{p}{q}$, where $p$ and $q$ are integers and $q$ is not zero. The rational numbers include zero and all the numbers that can be written as positive or negative fractions. They are the numbers you are familiar with from your previous work with numbers in arithmetic.

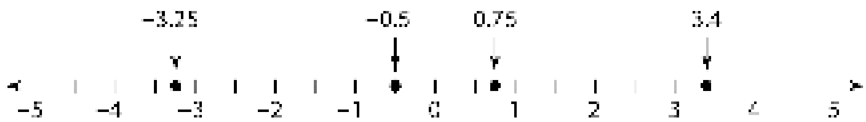
The rational numbers include whole numbers and integers. Here are examples.



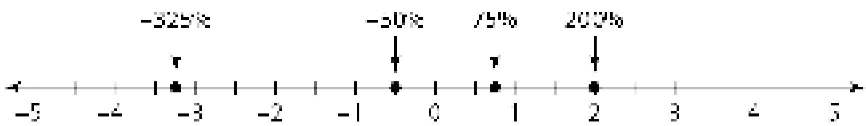

The rational numbers include positive and negative fractions. Here are examples.




The rational numbers include positive and negative repeating and terminating decimals. (See below for a discussion on repeating and terminating decimals.) Here are examples.



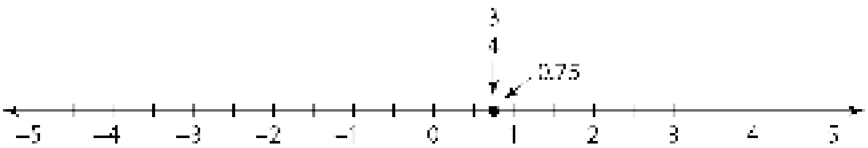

The rational numbers include positive and negative percents. Here are examples.



You obtain the equivalent decimal representation of a fraction, such as $\frac{3}{4}$, by performing the indicated division. (***Tip:*** Remember, $\frac{3}{4} = 3 \div 4$.) You divide the numerator by the denominator. Insert a decimal point in the numerator and zeros to the right of the decimal point to complete the division.

$$\begin{array}{r} 0.75 \\ 4\overline{)\ 3.00} \\ -2\,8\phantom{0} \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

The fraction $\frac{3}{4}$ and the **decimal** 0.75 are different representations of the same rational number. They are both located at the same location on the number line.




**Tip: If the fraction is a negative number, perform the division without the negative sign, and then attach the negative sign to the decimal expansion. For example, $-\frac{3}{4} = -0.75$.**

It is important you know the decimal expansion of a rational number either **terminates** in 0s or eventually **repeats**. In the case of $\frac{3}{4}$, you need to insert only two zeros after the decimal point for the division to finally reach a zero remainder. Inserting additional zeros would lead to repeated 0s to the right of 0.75 (like this: 0.75000…). You say that the decimal expansion of $\frac{3}{4}$ **terminates in 0s.**

However, for some rational numbers, the decimal expansion keeps going, but in a block of one or more digits that repeats over and over again. The repeating digits are not all zero. Here is an example.

$$\frac{5}{11} = 5 \div 11 = \begin{array}{r} 0.4545\ldots \\ 11\overline{)5.0000\ldots} \\ -4\,4\phantom{000\ldots} \\ \hline 60\phantom{00\ldots} \\ -55\phantom{00\ldots} \\ \hline 50\phantom{0\ldots} \\ -44\phantom{0\ldots} \\ \hline 60\phantom{\ldots} \\ -55\phantom{\ldots} \\ \hline \vdots\phantom{\ldots} \end{array}$$

No matter how long you continue to add zeros and divide, 45 in the quotient continues to repeat without end. You put a bar over one block of the repeating digits to indicate the repetition; thus, $\frac{5}{11} = 0.4545\ldots = 0.\overline{45}$. You say $\frac{5}{11}$ has a **repeating** decimal expansion. It is incorrect to write $\frac{5}{11} = 0.45$. Still, when decimals repeat, they are usually rounded to a specified degree of accuracy. For instance, $\frac{5}{11} \approx 0.45$ when *rounded* to two decimal places.

**Tip: Read the symbol ≈ as "is approximately equal to."**

Pretend you do not know 0.4545… is the decimal expansion of $\frac{5}{11}$. How would you go about converting the repeating decimal expansion 0.4545… into its fractional form $\frac{p}{q}$ $(q \neq 0)$? Here is the way to do it. (***Tip:*** This procedure works for any repeating decimal expansion.)

Let $x = 0.4545\ldots$. Do three steps. First, multiply both sides of the equation $x = 0.4545\ldots$ by $10^r$, where $r$ is the number of digits in the repeating block of digits in the decimal expansion. Next, subtract the original equation from the new equation. Then divide both sides of the resulting equation by the coefficient of $x$.

*Step 1.* Multiply both sides of the equation $x = 0.4545\ldots$ by $10^2 = 100$ (because two digits repeat).

$$\begin{aligned} x &= 0.4545\ldots \\ 100 \cdot x &= 100(0.4545\ldots) \\ 100x &= 45.4545\ldots \end{aligned}$$

*Step 2.* Subtract the original equation from the new equation.

$$\begin{array}{r} 100x = 45.4545\ldots \\ -x = -0.4545\ldots \\ \hline 99x = 45.0000\ldots \end{array}$$

*Step 3.* Solve for $x$ by dividing both sides of the resulting equation by the coefficient of $x$.

$$\begin{aligned} 99x &= 45 \\ \frac{\cancel{99}x}{\cancel{99}} &= \frac{45 \div 9}{99 \div 9} \\ x &= \frac{5}{11} \end{aligned}$$

Thus, $0.4545\ldots = \frac{5}{11}$.

***Tip:*** Notice when you multiply 0.4545… by 100, you can write the product as 45.4545…. You can do this because there are infinitely many 45s to the right of the decimal point, so you can write as many as you please.

Here is an additional example of the procedure.

Convert −3.666… to its equivalent fractional form.

**Tip: If the number is negative, do the conversion without the negative sign, and then attach**

**the negative sign to the fractional representation.**

Let $x = 3.666\ldots$

*Step 1.* Multiply both sides of the equation $x = 3.666\ldots$ by $10^1 = 10$ (because one digit repeats).

$$\begin{aligned} x &= 3.666\ldots \\ 10 \cdot x &= 10(3.666\ldots) \\ 10x &= 36.666\ldots \end{aligned}$$

*Step 2.* Subtract the original equation from the new equation.

$$\begin{array}{r} 10x = 36.666\ldots \\ -x = -3.666\ldots \\ \hline 9x = 33.000\ldots \end{array}$$

*Step 3.* Solve for $x$ by dividing both sides of the resulting equation by the coefficient of $x$.

$$\begin{aligned} 9x &= 33 \\ \frac{\cancel{9}x}{\cancel{9}} &= \frac{33 \div 3}{9 \div 3} \\ x &= \frac{11}{3} = 3\frac{2}{3} \end{aligned}$$

**Thus,** $3.666\ldots = \frac{11}{3} = 3\frac{2}{3}$.

## ☞ Try These

1. Fill in the blank(s).

   **(a)** A rational number is any number that can be expressed as                , where $p$ and $q$ are integers and $q$ is not zero.

   **(b)** The decimal expansion of a rational number either ___________ in 0s or eventually          .

2. Write the decimal expansion for the rational number.

   **(a)** $\frac{3}{8}$

   **(b)** $\frac{3}{7}$

   **(c)** $-\frac{2}{3}$

   **(d)** $\frac{7}{9}$

   **(e)** $-\frac{13}{9}$

3. Convert the decimal expansion to an equivalent fractional form.

(a) 0.666…

(b) 0.142857142857…

(c) 1.1818…

## Solutions

1.

(a) $\frac{p}{q}$

(b) terminates; repeats

2.

(a) $\frac{3}{8} = 0.375$

(b) $\frac{2}{7} = 0.\overline{285714}$

(c) $-\frac{2}{3} = -0.\bar{6}$

(d) $\frac{7}{9} = 0.\bar{7}$

(e) $-\frac{13}{9} = -1.\bar{4}$

3.

(a) $0.666\ldots = \frac{2}{3}$

$$
\begin{aligned}
x &= 0.666\ldots \\
10x &= 6.666\ldots \\
\underline{-x} &\underline{= -0.666\ldots} \\
9x &= 6.000\ldots \\
\frac{9x}{9} &= \frac{6}{9} \\
\frac{\cancel{9}x}{\cancel{9}} &= \frac{6 \div 3}{9 \div 3} \\
x &= \frac{2}{3}
\end{aligned}
$$

(b) $0.142857142857\ldots = \frac{1}{7}$

$$
\begin{aligned}
x &= 0.142857142857\ldots \\
1{,}000{,}000x &= 142{,}857.142857142857\ldots \\
\underline{\quad x} &= \underline{\quad\quad 0.142857142857\ldots} \\
999{,}999x &= 142{,}857 \\
\frac{999{,}999x}{999{,}999} &= \frac{142{,}857}{999{,}999} \\
\frac{\cancel{999{,}999}x}{\cancel{999{,}999}} &= \frac{142{,}857 \div 142{,}857}{999{,}999 \div 142{,}857} \\
x &= \frac{1}{7}
\end{aligned}
$$

**(c)** $1.1818\ldots = \frac{13}{11}$

$$
\begin{aligned}
x &= 1.1818\ldots \\
100x &= 118.1818\ldots \\
\underline{\quad x} &= \underline{\quad 1.1818\ldots} \\
99x &= 117 \\
\frac{99x}{99} &= \frac{117}{99} \\
\frac{\cancel{99}x}{\cancel{99}} &= \frac{117 \div 9}{99 \div 9} \\
x &= \frac{13}{11}
\end{aligned}
$$

# Recognizing Rational and Irrational Numbers

## (CCSS.Math.Content.8.NS.A.1)

**Irrational numbers** are numbers that cannot be written in the form $\frac{p}{q}$, where $p$ and $q$ are integers and $q$ is not zero. They have nonterminating, nonrepeating decimal expansions. An example of an irrational number is the positive number that multiplies by itself to give 2. This number is the principal square root of 2. Every positive number has two square roots: a positive square root and a negative square root. The positive square root is the **principal square root.** The square root symbol ($\sqrt{\ }$) is used to show the principal square root. Thus, the principal square root of 2 is written like this: $\sqrt{2}$. The other square root of 2 is $-\sqrt{2}$. It also is an irrational number.

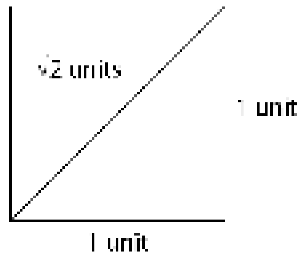
**Tip: Zero has only one square root; namely, zero (which is a rational number).**

You cannot express $\sqrt{2}$ as $\frac{p}{q}$, where $p$ and $q$ are integers ($q \neq 0$), and you cannot express it precisely in decimal form. No matter how many decimal places you use, you can only approximate $\sqrt{2}$. If you use a calculator to find the square root of the number 2, the display will show a decimal approximation of $\sqrt{2}$. An approximation of $\sqrt{2}$ to nine decimal places is 1.414213562. You can check whether this is $\sqrt{2}$ by multiplying it by itself to see whether you get 2.

$$1.414213562 \cdot 1.414213562 = 1.999999999$$

The number 1.999999999 is very close to 2, but it is not equal to 2. Only $\sqrt{2}$ or $-\sqrt{2}$ will multiply by itself to give 2. Thus, $\sqrt{2}\cdot\sqrt{2}=2$ and $-\sqrt{2}\cdot-\sqrt{2}=2$.

Even though an exact value for $\sqrt{2}$ cannot be determined, $\sqrt{2}$ is a number that occurs frequently in the real world. For instance, architects, carpenters, and other builders encounter $\sqrt{2}$ when they measure the length of the diagonal of a square that has sides with lengths of one unit, as shown here.




The diagonal of such a square measures $\sqrt{2}$ units.

**Tip: For most purposes, you can use 1.414 as an approximation for $\sqrt{2}$.**

There are an infinite number of square roots that are irrational. Here are a few examples.

$$\sqrt{3},\ \sqrt{10},\ -\sqrt{34},\ \sqrt{41},\ -\sqrt{89}$$

Another important irrational number is the number represented by the symbol $\pi$ (pi). The number $\pi$ also occurs frequently in the real world. For instance, $\pi$ is the number you get when you divide the circumference of a circle by its diameter. The number $\pi$ cannot be expressed as a fraction, nor can it be written as a terminating or repeating decimal. Here is an approximation of $\pi$ to nine decimal places: 3.141592654.

**Tip: There is no pattern to the digits of $\pi$. In this book, use the rational number 3.14 as an approximation for the irrational number $\pi$ in problems involving $\pi$.**

Not all square roots are irrational. For example, the principal square root of 25, denoted $\sqrt{25}$, is not irrational because $\sqrt{25}=5$, which is a rational number. The number 25 is a **perfect square** because its square root is rational. When you want to find the principal square root of a number, try to find a *nonnegative* number that multiplies by itself to give the number. You will find it helpful to memorize the following principal square roots.

| | | | | |
|---|---|---|---|---|
| $\sqrt{1}=1$ | $\sqrt{25}=5$ | $\sqrt{81}=9$ | $\sqrt{169}=13$ | $\sqrt{289}=17$ |
| $\sqrt{4}=2$ | $\sqrt{36}=6$ | $\sqrt{100}=10$ | $\sqrt{196}=14$ | $\sqrt{400}=20$ |
| $\sqrt{9}=3$ | $\sqrt{49}=7$ | $\sqrt{121}=11$ | $\sqrt{225}=15$ | $\sqrt{625}=25$ |
| $\sqrt{16}=4$ | $\sqrt{64}=8$ | $\sqrt{144}=12$ | $\sqrt{256}=16$ | |

Make yourself a set of flash cards or make matching cards for a game of "Memory." For the Memory game, turn all the cards facedown. Turn up two cards at a time. If they match (for instance $\sqrt{144}$ and 12 are a match), remove the two cards; otherwise, turn them facedown again. Repeat until you have matched all the cards.

Here are examples of rational square roots.

$-\sqrt{400}$, which is $-20$

$\sqrt{\frac{36}{49}}$, which is $\frac{6}{7}$

$\sqrt{\frac{100}{25}}$, which is $\sqrt{4} = 2$

$-\sqrt{\frac{9}{4}}$, which is $-\frac{3}{2}$

Keep in mind, though, every positive number has <u>two</u> square roots. The two square roots are equal in absolute value, but opposite in sign. For instance, the two square roots of 25 are 5 and –5, with 5 being the *principal* square root. Still, the square root symbol ($\sqrt{\ }$) *always* gives just <u>one</u> square root as the answer, and that square root is either positive or zero. Thus, $\sqrt{25} = 5$, not 5 or ±5 (read "plus or minus 5"). If you want ±5, then do this: $-\sqrt{25} = -5$.

**Tip: Recall that the absolute value of a specific number is just the value of the number with no sign attached.**

## ☞ Try These

1. Fill in the blank(s).

   **(a)** Irrational numbers are numbers that __________ (can, cannot) be written in the form $\frac{p}{q}$, where $p$ and $q$ are integers and $q$ is not zero.

   **(b)** Irrational numbers have nonterminating, __________ decimal expansions.

   **(c)** The square root symbol ($\sqrt{\ }$) is used to show a __________ square root, which is just one number.

   **(d)** Every positive number has __________ square roots.

   **(e)** Zero has __________ square root(s).

2. Indicate whether the number is rational or irrational by writing "rational" or "irrational" as your answer.

   **(a)** $\sqrt{88}$

   **(b)** $\sqrt{200}$

   **(c)** $\sqrt{81}$

   **(d)** $\sqrt{40}$

   **(e)** $3.\overline{45}$

   **(f)** 62.75

   **(g)** $\sqrt{\frac{25}{36}}$

(h) $-\sqrt{\frac{9}{16}}$

(i) $\pi$

**Solutions**

1.

(a) cannot
(b) nonrepeating
(c) principal
(d) two
(e) one

2.

(a) irrational
(b) irrational
(c) rational
(d) irrational
(e) rational
(f) rational
(g) rational
(h) rational
(i) irrational

# Approximating Irrational Numbers and Expressions

(CCSS.Math.Content.8.NS.A.2)

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You cannot write an exact decimal representation of an irrational square root. However, you can approximate its value to a desired number of decimal places. Here is an example.

Approximate $\sqrt{40}$ to the nearest hundredth.

*Step 1.* Approximate $\sqrt{40}$ to the nearest whole number.

Find two consecutive integers such that the square of the first integer is less than 40 and the square of the second integer is greater than 40. You know 6 × 6 = 36, which is less than 40, and 7 × 7 = 49, which is greater than 40. Thus, $\sqrt{36} < \sqrt{40} < \sqrt{49}$. So, the approximate value of $\sqrt{40}$ is between 6 and 7. It is closer to 6 because 40 is closer to 36 (4 units away) than it is to 49 (9 units away). To the nearest whole number, $\sqrt{40}$ is approximately 6.

*Step 2.* Approximate $\sqrt{40}$ to the nearest tenth.

Consider that 49 and 36 are 13 units apart and 40 and 36 are 4 units apart. So, 40 is $\frac{4}{13}$ of the distance between 49 and 36. As a rough approximation, $\sqrt{40}$ is about $\frac{4}{13}$ of the distance between 6 and 7. The distance between 6 and 7 is 1 unit. So, $\sqrt{40} \approx 6+\frac{4}{13}(1) \approx 6+0.307 \approx 6.3$. This calculation leads to the guess that $\sqrt{40}$ is between 6.3 and 6.4. Square each of these numbers, $6.3 \times 6.3 = 39.69$ and $6.4 \times 6.4 = 40.96$. The value of $\sqrt{40}$ is closer to 6.3 than it is to 6.4 because 40 is closer to 36.69 (0.31 unit away) than it is to 40.96 (0.96 unit away). To the nearest tenth, $\sqrt{40}$ is approximately 6.3.

*Step 3.* Approximate $\sqrt{40}$ to the nearest hundredth.

Consider that 40.96 and 39.69 are 1.27 units apart and 40 and 39.69 are 0.31 unit apart. So. 40 is $\frac{0.31}{1.27}$ of the distance between 40.96 and 39.69. As a rough approximation, $\sqrt{40}$ is about $\frac{0.31}{1.27}$ of the distancebetween 6.3 and 6.4. The distance between 6.3 and 6.4 is 0.1 unit. So, to the nearest hundredth, $\sqrt{40} \approx 6.3+\frac{0.31}{1.27}(0.1) \approx 6.3+0.024 \approx 6.32$. (Checking it, $6.32 \times 6.32 = 39.9424$, which is very close to 40.) On a number line, you would mark $\sqrt{40}$ at approximately 6.32.


To approximate irrational expressions, approximate the irrational component and then evaluate.

Here are examples.

Approximate $\pi^2$ to the nearest tenth.

$$\pi^2 \approx (3.14)^2 = 9.8596 \approx 9.9$$

Approximate $5\sqrt{2}$ to the nearest hundredth.

$$5\sqrt{2} \approx 5(1.414) = 7.07$$

## ☞ Try These

1. Approximate the irrational square root to the nearest tenth.
   - (a) $\sqrt{5}$
   - (b) $\sqrt{38}$
   - (c) $\sqrt{80}$
2. Using the results from question 1, approximate the irrational expression to the nearest tenth.
   - (a) $3\sqrt{5}$
   - (b) $8\sqrt{38}$

(c) $\frac{\sqrt{80}}{10}$

## Solutions

1.

(a) $\sqrt{5} \approx 2.2$

*Step 1.* Approximate . to the nearest whole number.

Find two consecutive integers such that the square of the first integer is less than 5 and the square of the second integer is greater than 5. You know 2 × 2 = 4, which is less than 5, and 3 × 3 = 9, which is greater than 5. Thus, $\sqrt{4} < \sqrt{5} < \sqrt{9}$. So, the approximate value of $\sqrt{5}$ is between 2 and 3. It is closer to 2 because 5 is closer to 4 (1 unit away) than it is to 9 (4 units away). To the nearest whole number, $\sqrt{5}$ is approximately 2.

*Step 2.* Approximate $\sqrt{5}$ to the nearest tenth.

Consider that 9 and 4 are 5 units apart and 5 and 4 are 1 unit apart. So, 5 is $\frac{1}{5}$ of the distance between 9 and 4. As a rough approximation, $\sqrt{5}$ is about $\frac{1}{5}$ of the distance between 2 and 3. The distance between 2 and 3 is 1 unit. So, $\sqrt{5} \approx 2 + \frac{1}{5}(1) = 2 + 0.2 = 2.2$. This calculation leads to the guess that $\sqrt{5}$ is between 2.2 and 2.3. Square each of these numbers, 2.2 × 2.2 = 4.84 and 2.3 × 2.3 = 5.29. The value of $\sqrt{5}$ is closer to 2.2 than it is to 2.3 because 5 is closer to 4.84 (0.16 unit away) than it is to 5.29 (0.29 unit away). To the nearest tenth, $\sqrt{5}$ is approximately 2.2.

(b) $\sqrt{28} \approx 5.3$

*Step 1.* Approximate $\sqrt{28}$ to the nearest whole number.

Find two consecutive integers such that the square of the first integer is less than 28 and the square of the second integer is greater than 28. You know 5 × 5 = 25, which is less than 28, and 6 × 6 = 36, which is greater than 28. Thus, $\sqrt{25} < \sqrt{28} < \sqrt{36}$. So, the approximate value of $\sqrt{28}$ is between 5 and 6. It is closer to 5 because 28 is closer to 25 (3 units away) than it is to 36 (8 units away). To the nearest whole number, $\sqrt{28}$ is approximately 5.

*Step 2.* Approximate $\sqrt{28}$ to the nearest tenth.

Consider that 36 and 25 are 11 units apart and 28 and 25 are 3 units apart. So, 28 is $\frac{3}{11}$ of the distance between 36 and 25. As a rough approximation, $\sqrt{28}$ is about $\frac{3}{11}$ of the distance between 5 and 6. The distance between 5 and 6 is 1 unit. So, $\sqrt{28} \approx 5 + \frac{3}{11}(1) \approx 5 + 0.272 \approx 5.3$. This calculation leads to theguess that $\sqrt{28}$ is between 5.3 and 5.4. Square each of these numbers, 5.3 × 5.3 = 28.09 and 5.4 × 5.4 = 29.16. The value of $\sqrt{28}$ is closer to 5.3 than it is to 6.4 because 28 is closer to 28.09 (0.09 unit away) than it is to 29.16 (1.16 units away). To the nearest tenth, $\sqrt{28}$ is approximately 5.3.

(c) $\sqrt{80} \approx 8.9$

*Step 1.* Approximate $\sqrt{80}$ to the nearest whole number.

Find two consecutive integers such that the square of the first integer is less than 80 and the