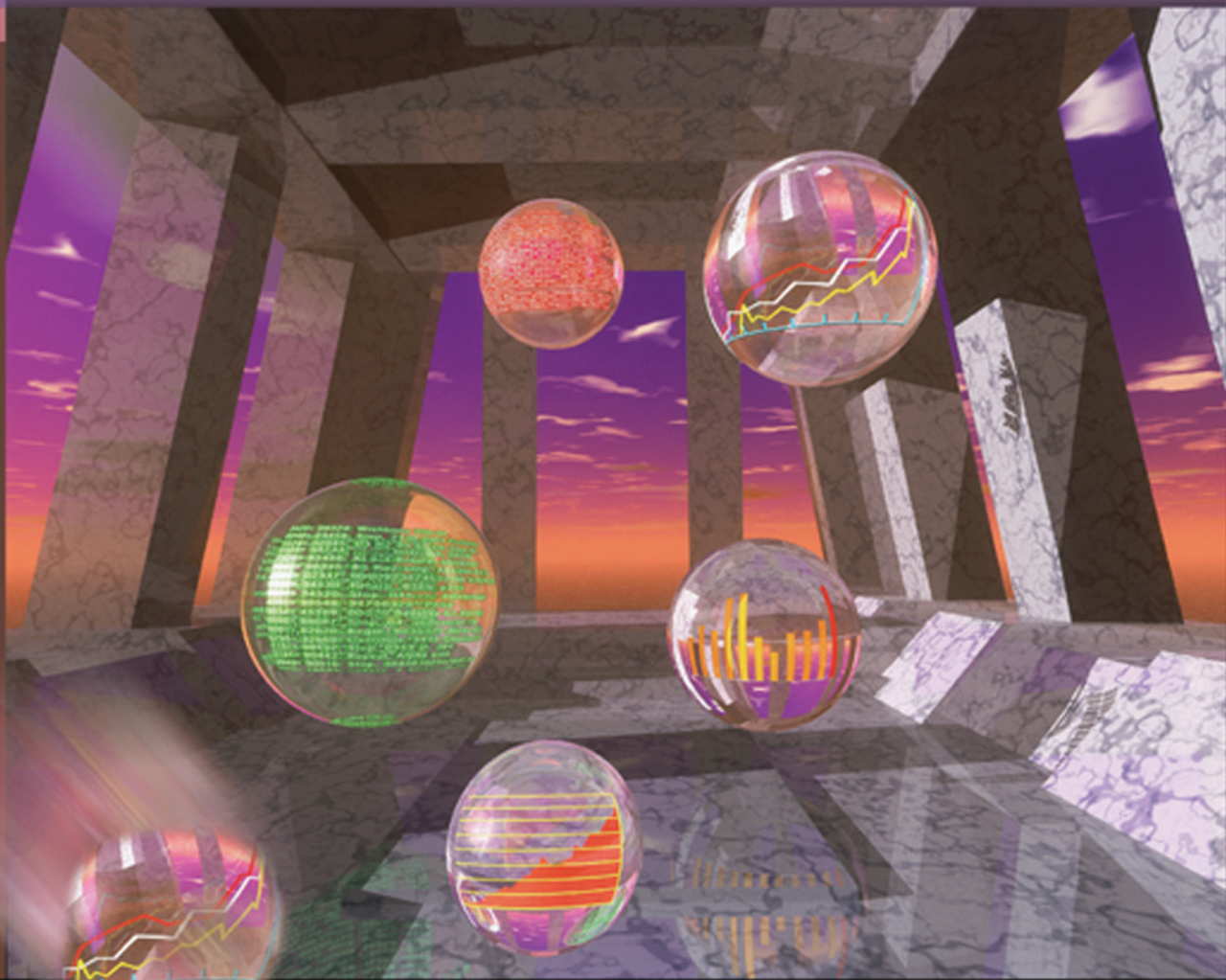




# Advanced Statistics from an Elementary Point of View

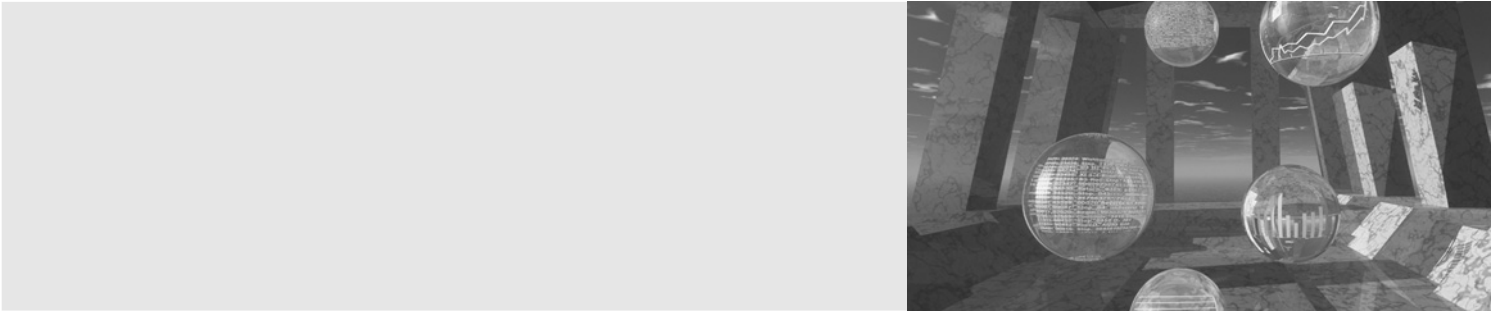


**MICHAEL J. PANIK**

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## **Advanced Statistics from an Elementary Point of View**





# Advanced Statistics from an Elementary Point of View

Michael J. Panik

University of Hartford



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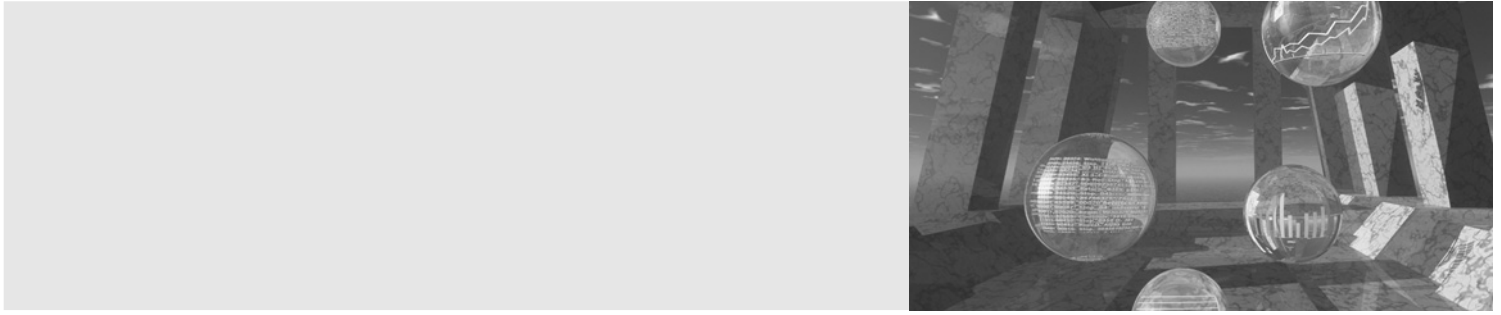
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To the Memory of  
Frank C. Grella  
Friend and Colleague





# Contents

## Preface xv

## 1 Introduction 1

- 1.1 Statistics Defined 1
- 1.2 Types of Statistics 1
- 1.3 Levels of Discourse: Sample vs. Population 2
- 1.4 Levels of Discourse: Target vs. Sampled Population 4
- 1.5 Measurement Scales 5
- 1.6 Sampling and Sampling Errors 7
- 1.7 Exercises 7

## 2 Elementary Descriptive Statistical Techniques 9

- 2.1 Summarizing Sets of Data Measured on a Ratio or Interval Scale 9
- 2.2 Tabular Methods 11
- 2.3 Quantitative Summary Characteristics 16
  - 2.3.1 Measures of Central Location 16
  - 2.3.2 Measures of Dispersion 21
  - 2.3.3 Standardized Variables 26
  - 2.3.4 Moments 29
  - 2.3.5 Skewness and Kurtosis 31
  - 2.3.6 Relative Variation 33
  - 2.3.7 Comparison of the Mean, Median, and Mode 34
  - 2.3.8 The Sample Variance and Standard Deviation 35
- 2.4 Correlation between Variables  $X$  and  $Y$  38
- 2.5 Rank Correlation between Variables  $X$  and  $Y$  42
- 2.6 Exercises 46

## 3 Probability Theory 53

- 3.1 Mathematical Foundations: Sets, Set Relations, and Functions 53
- 3.2 The Random Experiment, Events, Sample Space, and the Random Variable 59



3.3	Axiomatic Development of Probability Theory	62
3.4	The Occurrence and Probability of an Event	64
3.5	General Addition Rule for Probabilities	65
3.6	Joint, Marginal, and Conditional Probability	66
3.7	Classification of Events	72
3.8	Sources of Probabilities	77
3.9	Bayes' Rule	79
3.10	Exercises	82

## 4 Random Variables and Probability Distributions 93

4.1	Random Variables	93
4.2	Discrete Probability Distributions	94
4.3	Continuous Probability Distributions	101
4.4	Mean and Variance of a Random Variable	106
4.5	Chebyshev's Theorem for Random Variables	111
4.6	Moments of a Random Variable	113
4.7	Quantiles of a Probability Distribution	117
4.8	Moment-Generating Function	119
4.9	Probability-Generating Function	127
4.10	Exercises	132

## 5 Bivariate Probability Distributions 147

5.1	Bivariate Random Variables	147
5.2	Discrete Bivariate Probability Distributions	147
5.3	Continuous Bivariate Probability Distributions	154
5.4	Expectations and Moments of Bivariate Probability Distributions	162
5.5	Chebyshev's Theorem for Bivariate Probability Distributions	169
5.6	Joint Moment-Generating Function	169
5.7	Exercises	174

## 6 Discrete Parametric Probability Distributions 187

6.1	Introduction	187
6.2	Counting Rules	188
6.3	Discrete Uniform Distribution	194
6.4	The Bernoulli Distribution	195
6.5	The Binomial Distribution	197
6.6	The Multinomial Distribution	203
6.7	The Geometric Distribution	206
6.8	The Negative Binomial Distribution	208
6.9	The Poisson Distribution	212
6.10	The Hypergeometric Distribution	218
6.11	The Generalized Hypergeometric Distribution	225
6.12	Exercises	226

**7 Continuous Parametric Probability Distributions 235**

- 7.1 Introduction 235
- 7.2 The Uniform Distribution 236
- 7.3 The Normal Distribution 238
  - 7.3.1 Introduction to Normality 238
  - 7.3.2 The Z Transformation 240
  - 7.3.3 Moments, Quantiles, and Percentage Points 249
  - 7.3.4 The Normal Curve of Error 253
- 7.4 The Normal Approximation to Binomial Probabilities 253
- 7.5 The Normal Approximation to Poisson Probabilities 257
- 7.6 The Exponential Distribution 258
  - 7.6.1 Source of the Exponential Distribution 258
  - 7.6.2 Features/Uses of the Exponential Distribution 260
- 7.7 Gamma and Beta Functions 264
- 7.8 The Gamma Distribution 266
- 7.9 The Beta Distribution 270
- 7.10 Other Useful Continuous Distributions 276
  - 7.10.1 The Lognormal Distribution 276
  - 7.10.2 The Logistic Distribution 279
- 7.11 Exercises 285

**8 Sampling and the Sampling Distribution of a Statistic 293**

- 8.1 The Purpose of Random Sampling 293
- 8.2 Sampling Scenarios 294
  - 8.2.1 Data Generating Process or Infinite Population 294
  - 8.2.2 Drawings from a Finite Population 299
- 8.3 The Arithmetic of Random Sampling 301
- 8.4 The Sampling Distribution of a Statistic 306
- 8.5 The Sampling Distribution of the Mean 308
  - 8.5.1 Sampling from an Infinite Population 309
  - 8.5.2 Sampling from a Finite Population 311
- 8.6 A Weak Law of Large Numbers 316
- 8.7 Convergence Concepts 319
- 8.8 A Central Limit Theorem 322
- 8.9 The Sampling Distribution of a Proportion 326
- 8.10 The Sampling Distribution of the Variance 333
- 8.11 A Note on Sample Moments 338
- 8.12 Exercises 342

**9 The Chi-Square, Student's  $t$ , and Snedecor's  $F$  Distributions 349**

- 9.1 Derived Continuous Parametric Distributions 349
- 9.2 The Chi-Square Distribution 350

- 9.3 The Sampling Distribution of the Variance When Sampling from a Normal Population 354
- 9.4 Student's  $t$  Distribution 357
- 9.5 Snedecor's  $F$  Distribution 362
- 9.6 Exercises 368

## 10 Point Estimation and Properties of Point Estimators 373

- 10.1 Statistics as Point Estimators 373
- 10.2 Desirable Properties of Estimators as Statistical Properties 375
- 10.3 Small Sample Properties of Point Estimators 376
  - 10.3.1 Unbiased, Minimum Variance, and Minimum Mean Squared Error (MSE) Estimators 376
  - 10.3.2 Efficient Estimators 383
  - 10.3.3 Most Efficient Estimators 385
  - 10.3.4 Sufficient Statistics 394
  - 10.3.5 Minimal Sufficient Statistics 398
  - 10.3.6 On the Use of Sufficient Statistics 399
  - 10.3.7 Completeness 401
  - 10.3.8 Best Linear Unbiased Estimators 404
  - 10.3.9 Jointly Sufficient Statistics 405
- 10.4 Large Sample Properties of Point Estimators 408
  - 10.4.1 Asymptotic or Limiting Properties 408
  - 10.4.2 Asymptotic Mean and Variance 410
  - 10.4.3 Consistency 411
  - 10.4.4 Asymptotic Efficiency 416
  - 10.4.5 Asymptotic Normality 418
- 10.5 Techniques for Finding Good Point Estimators 419
  - 10.5.1 Method of Maximum Likelihood 419
  - 10.5.2 Method of Least Squares 430
- 10.6 Exercises 431

## 11 Interval Estimation and Confidence Interval Estimates 439

- 11.1 Interval Estimators 439
- 11.2 Central Confidence Intervals 441
- 11.3 The Pivotal Quantity Method 442
- 11.4 A Confidence Interval for  $\mu$  Under Random Sampling from a Normal Population with Known Variance 443
- 11.5 A Confidence Interval for  $\mu$  Under Random Sampling from a Normal Population with Unknown Variance 446
- 11.6 A Confidence Interval for  $\sigma^2$  Under Random Sampling from a Normal Population with Unknown Mean 447
- 11.7 A Confidence Interval for  $p$  Under Random Sampling from a Binomial Population 451

11.8	Joint Estimation of a Family of Population Parameters	455
11.9	Confidence Intervals for the Difference of Means When Sampling from Two Independent Normal Populations	458
11.9.1	Population Variances Known	461
11.9.2	Population Variances Unknown But Equal	461
11.9.3	Population Variances Unknown and Unequal	462
11.10	Confidence Intervals for the Difference of Means When Sampling from Two Dependent Populations: Paired Comparisons	464
11.11	Confidence Intervals for the Difference of Proportions When Sampling from Two Independent Binomial Populations	470
11.12	Confidence Interval for the Ratio of Two Variances When Sampling from Two Independent Normal Populations	471
11.13	Exercises	473

## 12 Tests of Parametric Statistical Hypotheses 483

12.1	Statistical Inference Revisited	483
12.2	Fundamental Concepts for Testing Statistical Hypotheses	484
12.3	What Is the Research Question?	486
12.4	Decision Outcomes	487
12.5	Devising a Test for a Statistical Hypothesis	488
12.6	The Classical Approach to Statistical Hypothesis Testing	491
12.7	Types of Tests or Critical Regions	493
12.8	The Essentials of Conducting a Hypothesis Test	495
12.9	Hypothesis Test for $\mu$ Under Random Sampling from a Normal Population with Known Variance	496
12.10	Reporting Hypothesis Test Results	501
12.11	Determining the Probability of a Type II Error $\beta$	504
12.12	Hypothesis Tests for $\mu$ Under Random Sampling from a Normal Population with Unknown Variance	510
12.13	Hypothesis Tests for $p$ Under Random Sampling from a Binomial Population	512
12.14	Hypothesis Tests for $\sigma^2$ Under Random Sampling from a Normal Population	516
12.15	The Operating Characteristic and Power Functions of a Test	519
12.16	Determining the Best Test for a Statistical Hypothesis	528
12.17	Generalized Likelihood Ratio Tests	537
12.18	Hypothesis Tests for the Difference of Means When Sampling from Two Independent Normal Populations	546
12.18.1	Population Variances Equal and Known	547
12.18.2	Population Variances Unequal But Known	547
12.18.3	Population Variances Equal But Unknown	548
12.18.4	Population Variances Unequal and Unknown	549
12.19	Hypothesis Tests for the Difference of Means When Sampling from Two Dependent Populations: Paired Comparisons	553

- 12.20 Hypothesis Tests for the Difference of Proportions When Sampling from Two Independent Binomial Populations 555
- 12.21 Hypothesis Tests for the Difference of Variances When Sampling from Two Independent Normal Populations 557
- 12.22 Hypothesis Tests for Spearman's Rank Correlation Coefficient  $\rho_S$  559
- 12.23 Exercises 561

### **13 Nonparametric Statistical Techniques 569**

- 13.1 Parametric vs. Nonparametric Methods 569
- 13.2 Tests for the Randomness of a Single Sample 572
- 13.3 Single-Sample Sign Test Under Random Sampling 580
- 13.4 Wilcoxon Signed Rank Test of a Median 583
- 13.5 Runs Test for Two Independent Samples 587
- 13.6 Mann-Whitney (Rank-Sum) Test for Two Independent Samples 590
- 13.7 The Sign Test When Sampling from Two Dependent Populations: Paired Comparisons 597
- 13.8 Wilcoxon Signed Rank Test When Sampling from Two Dependent Populations: Paired Comparisons 599
- 13.9 Exercises 603

### **14 Testing Goodness of Fit 609**

- 14.1 Distributional Hypotheses 609
- 14.2 The Multinomial Chi-Square Statistic: Complete Specification of  $H_0$  609
- 14.3 The Multinomial Chi-Square Statistic: Incomplete Specification of  $H_0$  616
- 14.4 The Kolmogorov-Smirnov Test for Goodness of Fit 621
- 14.5 The Lilliefors Goodness-of-Fit Test for Normality 630
- 14.6 The Shapiro-Wilk Goodness-of-Fit Test for Normality 631
- 14.7 The Kolmogorov-Smirnov Test for Goodness of Fit: Two Independent Samples 632
- 14.8 Assessing Normality via Sample Moments 634
- 14.9 Exercises 638

### **15 Testing Goodness of Fit: Contingency Tables 643**

- 15.1 An Extension of the Multinomial Chi-Square Statistic 643
- 15.2 Testing Independence 643
- 15.3 Testing  $k$  Proportions 649
- 15.4 Testing for Homogeneity 651
- 15.5 Measuring Strength of Association in Contingency Tables 655
- 15.6 Testing Goodness of Fit with Nominal-Scale Data: Paired Samples 661
- 15.7 Exercises 664

**16 Bivariate Linear Regression and Correlation 669**

16.1	The Regression Model	669
16.2	The Strong Classical Linear Regression Model	670
16.3	Estimating the Slope and Intercept of the Population Regression Line	673
16.4	Mean, Variance, and Sampling Distribution of the Least Squares Estimators $\hat{\beta}_0$ and $\hat{\beta}_1$	676
16.5	Precision of the Least Squares Estimators $\hat{\beta}_0, \hat{\beta}_1$ : Confidence Intervals	679
16.6	Testing Hypotheses Concerning $\beta_0, \beta_1$	680
16.7	The Precision of the Entire Least Squares Regression Equation: A Confidence Band	684
16.8	The Prediction of a Particular Value of $Y$ Given $X$	687
16.9	Decomposition of the Sample Variation of $Y$	691
16.10	The Correlation Model	695
16.11	Estimating the Population Correlation Coefficient $\rho$	697
16.12	Inferences about the Population Correlation Coefficient $\rho$	698
16.13	Exercises	705

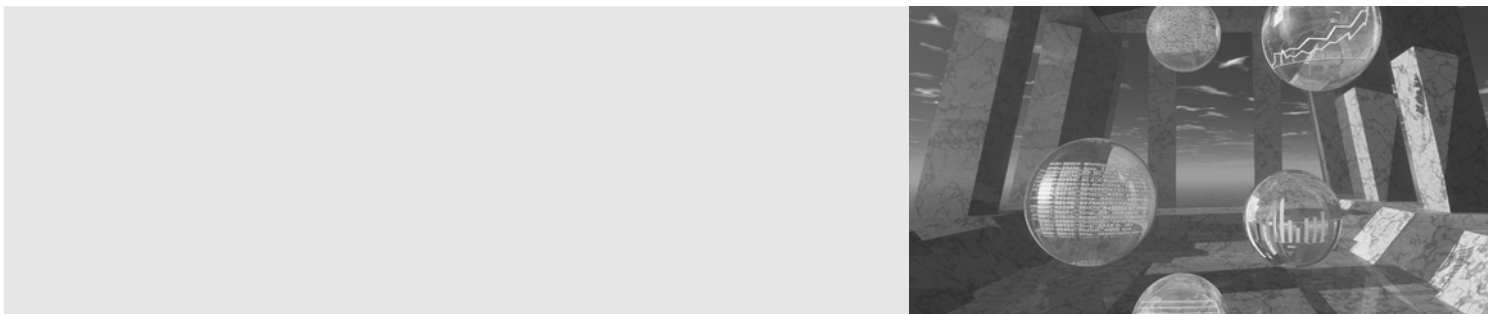
**Appendix A 717**

<b>Table A.1</b>	Standard Normal Areas	718
<b>Table A.2</b>	Cumulative Distribution Function Values for the Standard Normal Distribution	719
<b>Table A.3</b>	Quantiles of Student's $t$ Distribution	721
<b>Table A.4</b>	Quantiles of the Chi-Square Distribution	722
<b>Table A.5</b>	Quantiles of Snedecor's $F$ Distribution	724
<b>Table A.6</b>	Binomial Probabilities	727
<b>Table A.7</b>	Cumulative Distribution Function Values for the Binomial Distribution	733
<b>Table A.8</b>	Poisson Probabilities	738
<b>Table A.9</b>	Fisher's $\hat{\rho}(=r)$ to $\xi$ Transformation	744
<b>Table A.10</b>	$R$ Distribution for the Runs Test of Randomness	745
<b>Table A.11</b>	$W^+$ Distribution for the Wilcoxon Signed Rank Test	746
<b>Table A.12</b>	$R_1$ Distribution for the Mann-Whitney Rank-Sum Test	747
<b>Table A.13</b>	Quantiles of the Lilliefors Test Statistic $\hat{D}_n$	756
<b>Table A.14</b>	Quantiles of the Kolmogorov-Smirnov Test Statistic $D_n$	757
<b>Table A.15</b>	Quantiles of the Kolmogorov-Smirnov Test Statistic $D_{n,m}$ When $n = m$	758
<b>Table A.16</b>	Quantiles of the Kolmogorov-Smirnov Test Statistic $D_{n,m}$ When $n \neq m$	759
<b>Table A.17</b>	Quantiles of the Shapiro-Wilk Test Statistic $W$	761
<b>Table A.18</b>	Coefficients for the Shapiro-Wilk Test	762
<b>Table A.19</b>	Durbin-Watson $DW$ Statistic	764
<b>Table A.20</b>	$D$ Distribution of the von Neumann Ratio of the Mean Square Successive Difference to the Variance	766

**Solutions to Selected Exercises 767**

**References and Suggested Reading 785**

**Index 789**



## Preface

This book is intended as an introduction to probability and statistical inference for junior- or senior-level students in one- or two-semester courses offered by departments of mathematics or statistics. It can also serve as the foundation text for first-year graduate students in disciplines such as engineering, economics, and the physical and life sciences, where considerable use of statistics is the norm.

No previous study of probability or statistical inference is assumed. The only prerequisite is the standard introductory course in the calculus. Review sections dealing with set algebra, functions, and basic combinatorics are included for your convenience.

A strength of this book is that it is highly readable. Great care has been taken to fully develop statistical concepts and definitions. Detailed explanations of theorems, tests, and results are offered without compromising the rigor and integrity of the subject matter. An objective of this work is to get students to concentrate on the statistics without being overwhelmed by the calculations. Students who have used this book should be well on their way to thinking like a statistician when it comes to problem solving- and decision- making.

An important feature of this text is the considerable attention given to sampling distributions, point and interval estimation, parametric and distributional hypothesis testing, and linear regression and correlation. These topics typically constitute the heart and soul of most statistics courses, and this book has been written with this notion in mind.

This book can be used at a variety of levels. If theorem content but not theorem proof is important, then the general flow of the various chapters can be followed and the task-oriented/applications exercises found at the end of each chapter can be selectively chosen. However, if proofs and derivations are an integral part of the course, then the exercises that address the same can be attempted. The motivation underlying the execution of each proof as well as step-by-step details necessary for its completion are offered in the *Instructor's Manual*. So although the main text is certainly not devoid of proofs (it engages the reader in proofs that are more or less constructive or that reinforce the conceptual notions and definitions at hand), the more complex and mathematically challenging proofs



are available as standalone items and presented without impeding the continuity of presentation of the basic material.

After a review in Chapter 2 of some basic descriptive concepts, Chapter 3 develops the rudiments of probability theory. The latter is a key chapter since it sets the stage for the study of a broad range of inferential statistical techniques that follow. Chapter 4 treats general univariate probability distributions in considerable detail, and Chapter 5 does the same for general bivariate probability distributions. Chapters 6 and 7 introduce you to a variety of important specific discrete and continuous probability distributions, respectively. The bivariate normal distribution is also introduced in Chapter 7.

Chapter 8 exposes you to the concept of random sampling and the sampling distribution of a statistic (including those of the mean, proportion, and variance). Laws of large numbers and a Central Limit Theorem are carefully developed and explained. Chapter 9 deals with a set of derived distributions (chi-square,  $t$ , and  $F$ ) and revisits the sampling distribution of the variance under the normality assumption.

Point estimation is the topic of Chapter 10. Small-sample as well as large-sample properties of point estimators are covered along with a variety of techniques for finding good point estimators. This is a critical chapter in that you are introduced to the Cramér-Rao lower bound, the Fisher-Neyman Factorization Theorem, and the theorems of Lehmann-Scheffé and Rao-Blackwell. The methods of least squares, maximum likelihood, and best linear unbiased estimation are fully explored. The method of moments technique is also addressed in the chapter exercises.

Chapter 11 introduces you to the construction of a variety of single-sample and two-sample confidence intervals. Independent populations as well as paired comparisons are considered. In addition, the joint estimation of a family of population parameters is conducted using the Bonferroni method.

Parametric statistical hypothesis testing is the topic of Chapter 12. Great care is taken to develop the preliminaries. That is, issues such as statistical hypothesis formulation, the research question, varieties of decision outcomes, errors in testing, devising tests, types of tests, and so on, are treated in detail before any actual testing is undertaken. Determining the best test for a statistical hypothesis, the power of a test, and generalized likelihood ratio tests are included to complete the hypothesis testing methodology. Various one-sample and two-sample hypothesis tests are conducted, where the latter involve both independent and dependent populations. Hypothesis tests for Spearman's rank correlation coefficient round out the chapter. Throughout all of the presentation, the appropriate reporting of hypothesis testing results is emphasized.

Chapter 13 involves a collection of nonparametric hypothesis tests. Here, both single-sample and two-sample tests are executed. Comparisons between parametric and non-parametric tests are frequently made as successive tests are developed.

Testing goodness of fit is the thrust of Chapters 14 and 15. Chapter 14 treats distributional hypotheses (via chi-square Kolmogorov-Smirnov, Lilliefors, and

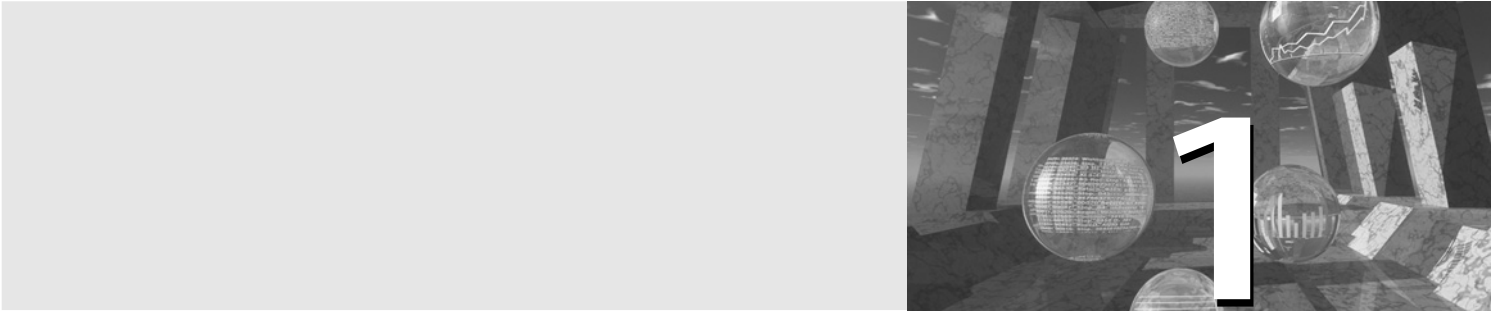
Shapiro-Wilk procedures) and Chapter 15 employs contingency tables to test for independence, homogeneity, and uniformity among a set of proportions. Issues concerning strength of association are also explored.

Chapter 16 offers an extremely detailed discussion of bivariate regression and correlation. Topics treated include the assumptions underlying the strong vs. weak classical linear regression models, the Gauss-Markov Theorem, least squares estimation, hypothesis test for the population parameters, confidence bands, prediction, decomposition of the sample variation in the dependent variable, the correlation model, and inferences about the population correlation coefficient. Embellishments of the basic regression model (e.g., dummy variables, nonlinearities, etc.) are found in the chapter exercises.

Many individuals have helped to make this work possible. A debt of gratitude is owed to each of them. First and foremost is my wife, Paula, whose support, patience, and encouragement helped sustain me throughout all the writing and rewriting. I would also like to thank the Department of Economics, Finance and Insurance, the Barney School, and the University's Coffin Grant Committee for financial assistance. Additionally, I am grateful to Alice Schoenrock and to a whole host of graduate assistants, particularly Amlan Datta, who helped with many aspects of the preparation of the basic manuscript and accompanying tables. Special accolades go to Marilyn Baleshiski who expertly typed the final draft of the entire manuscript.

A special note of thanks is extended to the Editorial and Production Departments at Elsevier. Acquisition Editors Barbara Holland and Tom Singer, along with the Project Manager, Sarah Hajduk, made the entire publication process quite painless. Their kindness and professionalism are deeply appreciated. Furthermore, the following reviewers generously offered many valuable comments about the manuscript: John Travis, Mississippi College; Laura McSweeney, Fairfield University; Eric Slud, University of Maryland at College Park; Tom Short, Indiana University of Pennsylvania; Cristopher Mechlin, Murray State University; Pierre Grillet, Tulane University; and Mohammad Shakil, Miami Dade University. I appreciate their insight and constructive suggestions.





# Introduction

## 1.1 Statistics Defined

Broadly defined, statistics encompasses the theory and methods of collecting, organizing, presenting, analyzing, and interpreting data sets so as to determine their essential characteristics. Although the collection, organization, and presentation of data will be addressed frequently throughout the text, primary emphasis will be placed upon the analysis of data and the interpretation of the results. Underlying the analysis of data is the vast mathematical apparatus of abstract concepts, theorems, formulae, algorithms, and so on that constitute the statistical tools that we will employ to study a data set. In this regard, our goal is to develop *a kit of tools* with which to analyze and interpret data. These tools will then enable us to build a framework for good decision making.

## 1.2 Types of Statistics

There are essentially two major categories of statistics: descriptive and inductive. *Descriptive statistics* includes any treatment of data designed to *summarize* their essential features. Here we are interested in arranging data in a readable form; for example, we may construct tables, charts, and graphs; and we can compute percents, averages, rates of change, and so on. In this regard, we do not go beyond the data at hand.

With *inductive statistics*, we are concerned with making estimates, predictions or forecasts, and generalizations. Since induction is the process of reasoning from the specific to the general, the essential characteristic of inductive statistics is termed *statistical inference*—the process of inferring something about *the whole* from an examination of only *the part*. Specifically, this process is carried out through *sampling*; that is, a representative subgroup of items is subject to study (the part) and the conclusions derived therefrom are assumed to characterize the entire group (the whole). Moreover, since an exhaustive *census* (or complete enumeration) of the whole is not being undertaken, so that our conclusions are hostage to the characteristics of those items actually comprising the part, some

level of error will most assuredly taint our conclusions about the whole. Hence we must accompany any generalization about the whole by a measure of the uncertainty of the inference made. Such measures can be calculated once the rudiments of probability theory are covered.

For instance, suppose we want to gain some insight into the relative popularity of a collection of candidates for the presidency of the United States. Can we realistically poll each and every individual of voting age (the whole)? Certainly not. But we can, using scientific polling techniques, elicit the preferences of only a small segment of all potential voters (the part). Hence, we may possibly conclude from this exercise that candidate A is the choice of 64% of *all* eligible voters. But this conclusion is not couched in absolute certainty. Some margin of error emerges since only a sample of individuals was taken. Hence we would accompany the 64% figure with a statement reading something like: the degree of precision of our estimate is  $\pm 3\%$  with a 95% reliability level. The notions of *precision* and *reliability* will play a key role in the development of our inferential techniques.

In sum, if we only want to summarize or present data or just catalog facts, then we will use descriptive statistics. But if we want to make inferences based on sample data or make decisions in the face of uncertainty, then we must rely on inductive statistical methods.

### 1.3 Levels of Discourse: Sample vs. Population

Let us further elaborate on the concepts of *the whole* and *the part*. To set the stage for this discussion let us define an *experiment* as any process of observation. It may involve something like quality control inspection of electric motors or simply monitoring the flow of various types of motor vehicles along a particular street on a given day and over a given time interval. Next, a *variable* (denoted as  $X$ ) can be thought of as any observable or measurable magnitude. Here  $X$  may depict an individual's set of exam scores, height (in inches) of sixth graders in a particular school system, weight (in pounds) of dressed turkeys, elapsed time (in minutes) taken to perform a certain task, and so on.

In this regard, let us define the sample (the part) as those things or events that both happened and were observed. It is drawn from the population (the whole), which involves things or events that happened but were not necessarily observed. Equivalently, we may think of the *population* as representing all conceivable observations on some variable  $X$ , whereas a *sample* is simply a subset of the population, that is, a given collection of observations taken from the population (see Figure 1.1). By convention, for a finite population, let us depict the population size by  $N$  and the sample size by  $n$ , where obviously  $n \leq N$ . Moreover, if  $X$  depicts a population variable, then the items within the population can be listed as  $X : X_1, X_2, X_3, \dots, X_N$  or as  $X : X_i, i = 1, \dots, N$ . And if  $X$  represents a sample variable, then the collection of sample items can be written as  $X : X_1, X_2, \dots, X_n$  or  $X : X_i, i = 1, \dots, n$ .

A sample is of finite size, but a population can be finite or infinite. That is, the collection of all individuals who have read part or all of this book can be thought

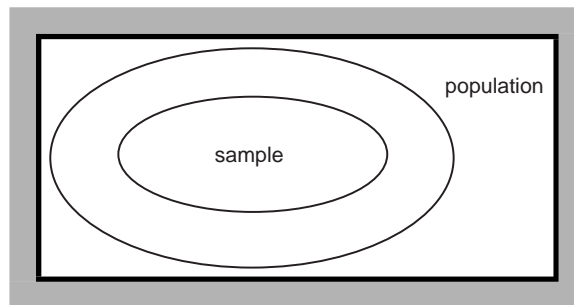


Figure 1.1 A sample as a subset of a population.

of as constituting a finite population, whereas an infinite population can easily be generated by sampling with replacement. For instance, if a population consists of  $N$  items and we *sample without replacement*, then the item obtained on any draw is set aside before the next item is chosen. But if we *sample with replacement* (on each draw we select an item from the population and then replace it before the next one is chosen), then clearly we can effectively sample in this fashion forever; that is, we can operate *as if* the population is infinite even though in reality it is not. So under sampling with replacement, a member of the population can appear more than once in the sample; in sampling without replacement, we disregard any item that has already been chosen for inclusion in the sample.

A few additional points merit our attention. First, it is important to mention that oftentimes inductive statistics is described as *decision making under uncertainty*. For our immediate purposes, note that an important source of uncertainty is the concept of *randomness*. That is, if the outcome of an experiment is not predictable, then it is said to occur in a random fashion. Next, we stated earlier that in sampling, a representative group of items is desired. In this regard, we may deem a sample as representative if it is *typical*, that is, it adequately reflects the attributes of the population. Moreover, we will frequently engage in the process of random sampling. Specifically, a sample is *random* if each and every item in the population has an equal (and known) chance of being included in the sample.

It is important to remember that the process of random sampling does not guarantee that a representative sample will be obtained. Randomization is likely but by no means certain to generate a representative sample. This is because randomization gives the same chance of selection to every sample of a given size—representative ones as well as nonrepresentative ones.

We end this section by noting that the term *parameter* (denoted  $\theta$ ) is used to represent any descriptive measure of a population, whereas a *statistic* (denoted  $\hat{\theta}$ ) is any descriptive measure of a sample. Here  $\hat{\theta}$  serves as an *estimator* of (the unknown)  $\theta$ ; that is,  $\hat{\theta}$  is some function of the sample values used to discern the level of  $\theta$ . The  $\hat{\theta}$  actually obtained by the estimation process will be called an *estimate* of  $\theta$ . If  $\hat{\theta}$  represents a single numerical value, then it is termed a *point estimator* of  $\theta$ . An *interval estimator* of  $\theta$  enables us to state just how confident

we are, in terms of probability, that  $\theta$  lies within some range of values. We shall return to these notions in Chapters 10 and 11.

## 1.4 Levels of Discourse: Target vs. Sampled Population

The *target population* is defined as the population to be studied; it is the population about which information is desired. This is in contrast to the *sampled population*—the population from which the sample is actually obtained. This latter population concept is alternatively called the *sampling frame*, or just *frame* for short. Based upon these two population notions, consequently we can describe a sample (or study for that matter) as being *valid* if the target and sampled populations have similar characteristics. (Note that some items in the target population may not be a member of the frame.)

Continuing in this vein, an *elementary sampling unit* is an item in the frame and an *observation* is a piece of information possessed by an elementary sampling unit. A *sample*, then, is that portion of the sampled population actually studied. (Note also that a sample may be representative—it adequately reflects the attributes of the frame—but not valid.) So under random sampling, each item in the frame has the same chance of being chosen.

For example, suppose we want to develop a profile of the membership of the local country club. This is the target population. The sampled population or frame is the club's membership list. If the list is up to date, then the target and sampled populations coincide. However, if the list has not been updated recently, then the target and sampled populations may be widely disparate and thus the question of validity comes to the fore. Elementary sampling units are the individual members, whereas an observation consists of a particular data point or value of some characteristic of interest. In what follows we shall depict each characteristic by a separate variable. Hence an observation is the value of the variable representing a characteristic of an elementary sampling unit.

For instance, assume that a country club has 3000 ( $= N$ ) members and that we want a sample of size 100 ( $= n$ ). If the membership list is arranged alphabetically (which we can assume to be a random arrangement), then we can easily engage in a sampling process called *systematic random sampling*, with a *sampling cycle* of  $30 = 3000/100 (= N/n)$ . That is, we start at the beginning of the list, select one member at random from the first 30 members listed, and then pick from that item on every thirtieth number for inclusion in the sample.

Which characteristic of a member (our elementary sampling unit) might we be interested in? We can possibly list them as sex, number of years as a member, number of years service as a board member, type of membership (e.g., founding, charter, regular), annual gift amount, major activity preferred (e.g., golf, tennis, swimming, bridge), and so on. As indicated earlier, each characteristic is represented by a variable and the value of the variable is an observation of the characteristic, as listed in Table 1.1.

Table 1.1

Country Club Membership Characteristics		
Characteristic	Variable	Observation Values
Sex	$X_1$	Male (1); Female (2)
Years	$X_2$	Number of years
Board	$X_3$	Number of years
Type	$X_4$	Founding (3); Charter (2); Regular (1)
Gift	$X_5$	\$
Activity	$X_6$	Golf (1); Tennis (2); Swimming (3); Bridge (4)

## 1.5 Measurement Scales

What types or varieties of data have we defined in the preceding example on country club membership characteristics? We may generally refer to *data* as a collection of facts, values, observations, or measurements. So if our data consists of observations that can be *measured* (i.e., classified, ordered, or quantified), then at what level does the measurement take place? Here we are interested in the *forms* in which data is found or the *scales* on which data is measured. These scales, stated in terms of increasing information content, are classified as nominal, ordinal, interval, and ratio.

Let us first consider the *nominal* scale. Here nominal should be associated with the word *name* since this scale identifies categories. Observations on a nominal scale possess neither numerical values nor order. However, observations on this type of scale can be given numerical codes such as “0 or 1” or “1, 2, 3, . . . .” Variables  $X_1$  and  $X_6$  in Table 1.1 are nominal in nature and are termed *qualitative* or *categorical* variables. Note that when dealing with a nominal scale, the categories defined must be *mutually exclusive* (each item falls into one and only one category) and *collectively exhaustive* (the list of categories is complete in that each item can be classified). Since  $X_1$  has only two categories, it is called a binary or *dichotomous* variable. Note also that a number code has been used to classify the members as either 1 (male) or 2 (female). These numbers serve only as *identifiers*; the magnitude of the differences between these numerical values is meaningless. The only valid operations for variables represented by a nominal scale are the determination of “=” or “≠.”

The *ordinal* scale (think of the word *order*) includes all properties of the nominal scale with the additional property that the observations can be ranked from the smallest to the largest or from the least important to the most important. (Note that nominal measurements cannot be ordered—all items are treated equally.) If the country club mentioned earlier is a hierarchical organization wherein founding members are more important or ranked higher (in terms of privileges) than charter members, which are in turn ranked above regular members, then  $X_4$  is an ordinal (and thus qualitative) variable. (Although chartered is in some sense *better* than regular, the ranking does not indicate *how much better*.) That is, since the numerical values assigned to  $X_4$  are 3, 2, and 1, these numbers



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